The Public University Secretary Problem

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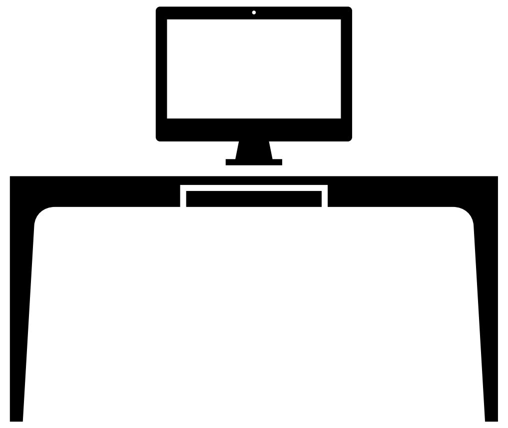
SOSA 2024

Joint work with: Benjamin Moseley (Carnegie Mellon) and Kirk Pruhs (U. of Pittsburgh)

“RAND” (Classic) Secretary Problem



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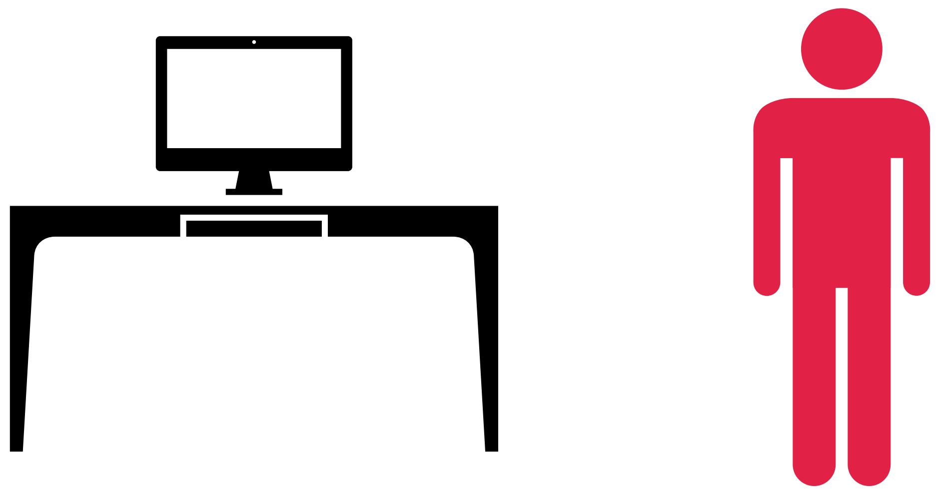

$x_1 = 4$

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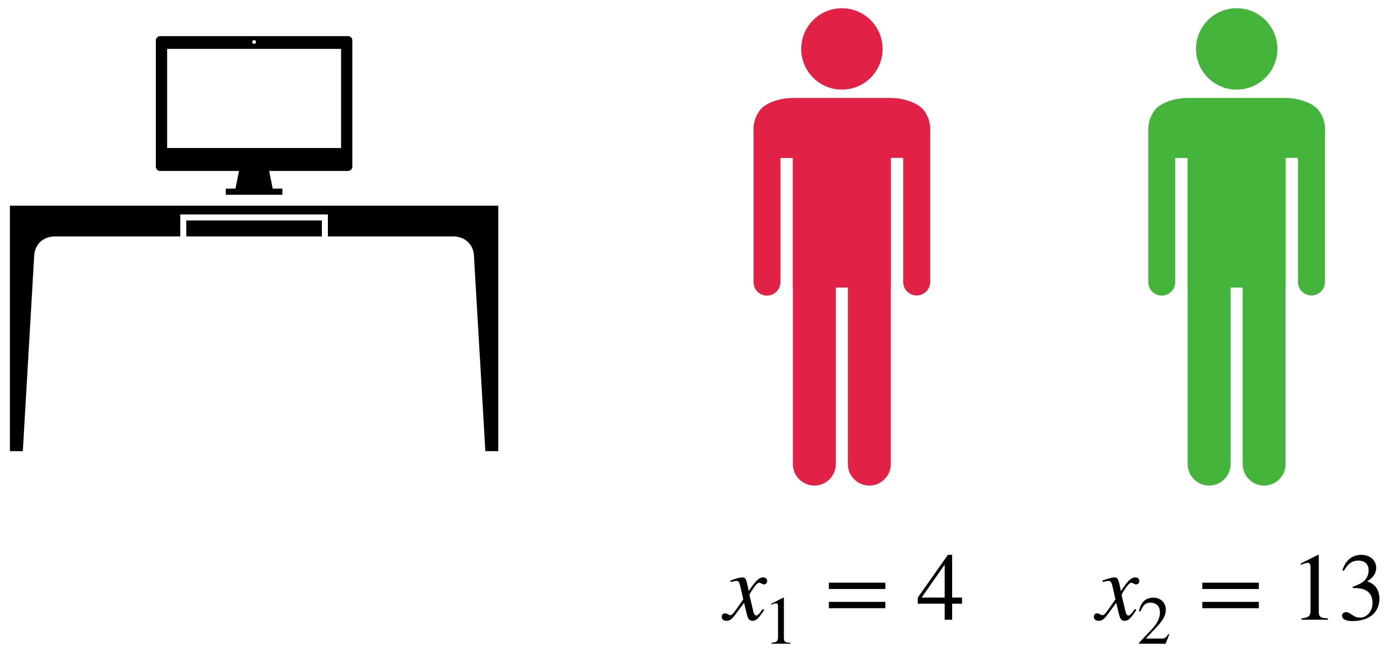

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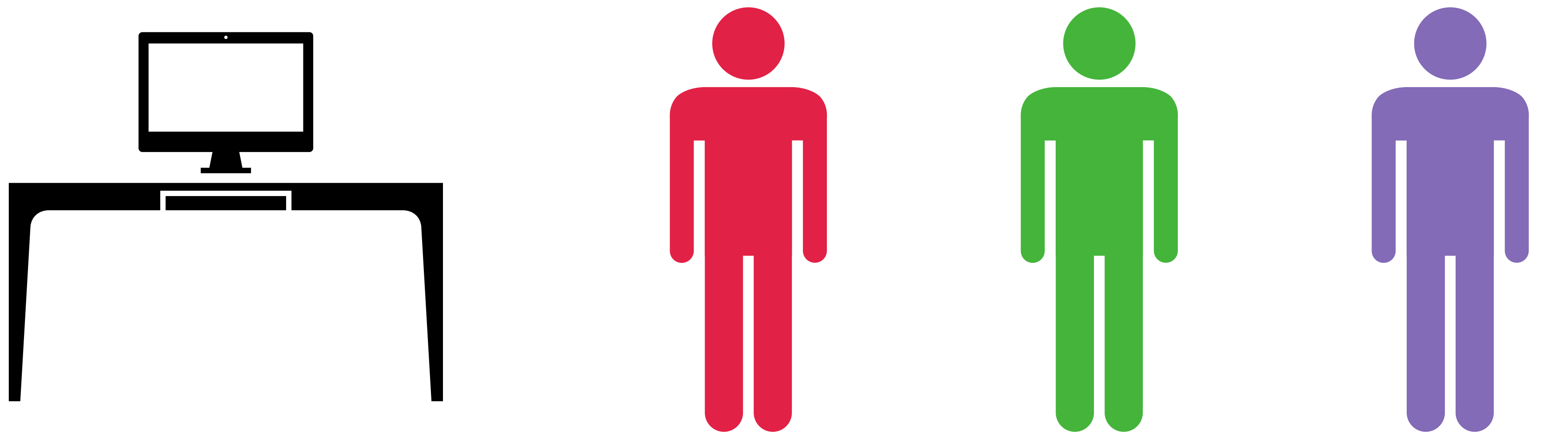

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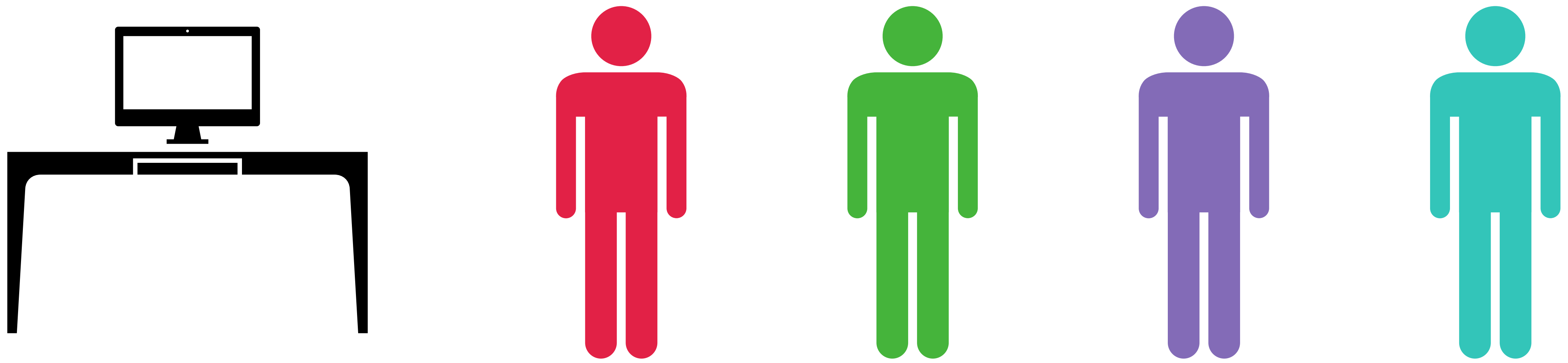

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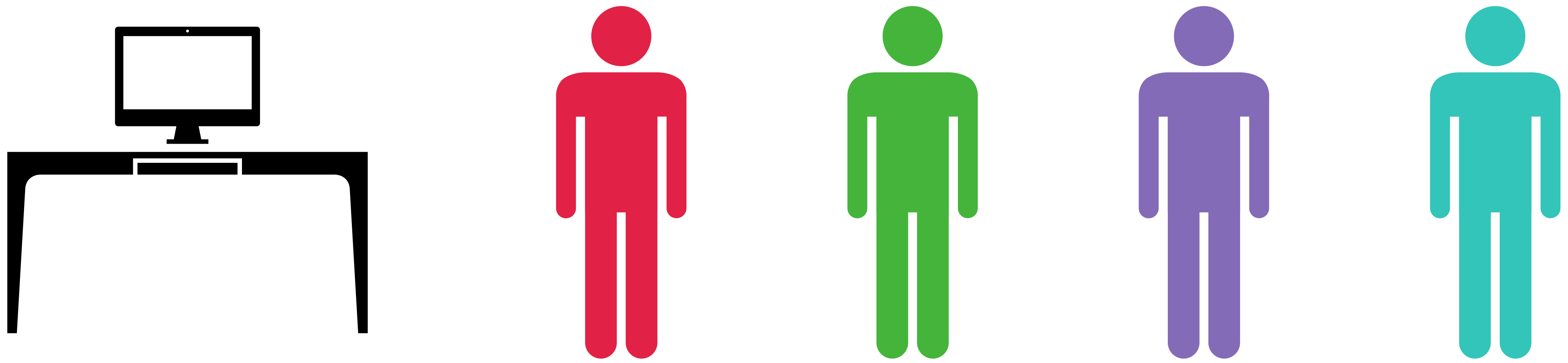

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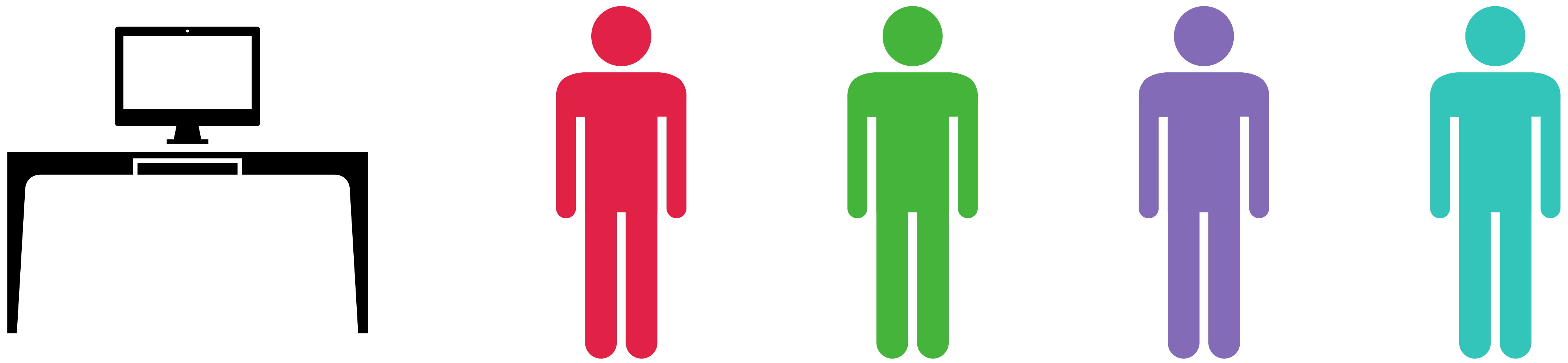

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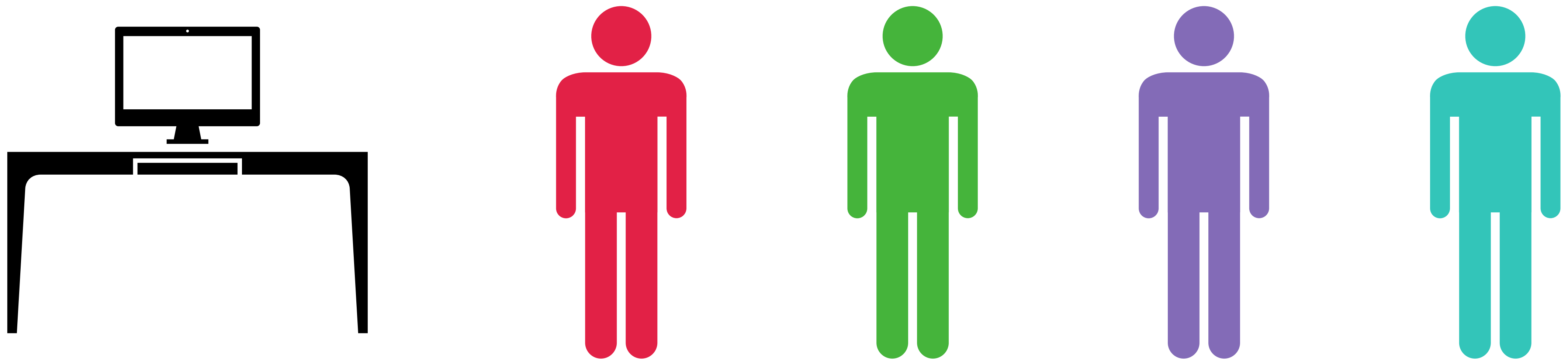

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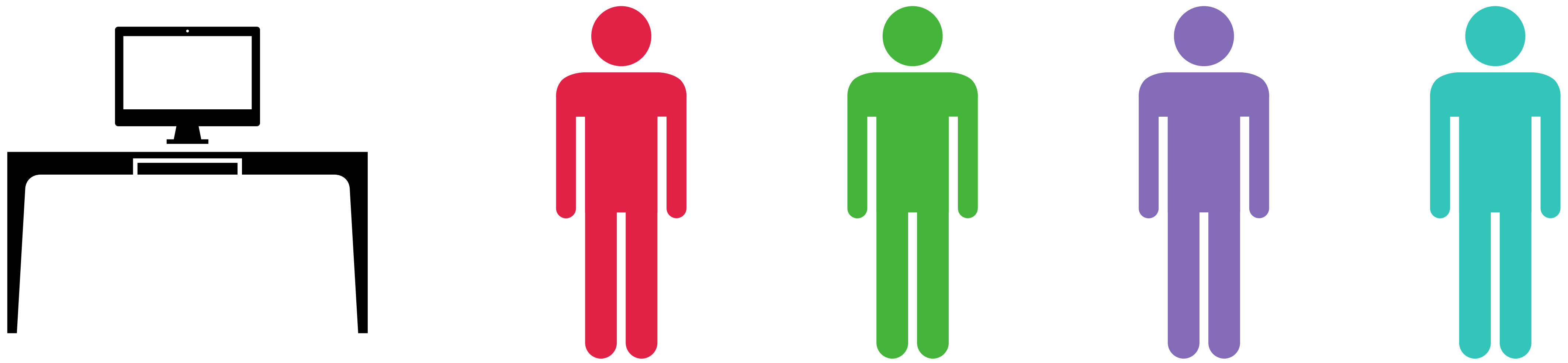

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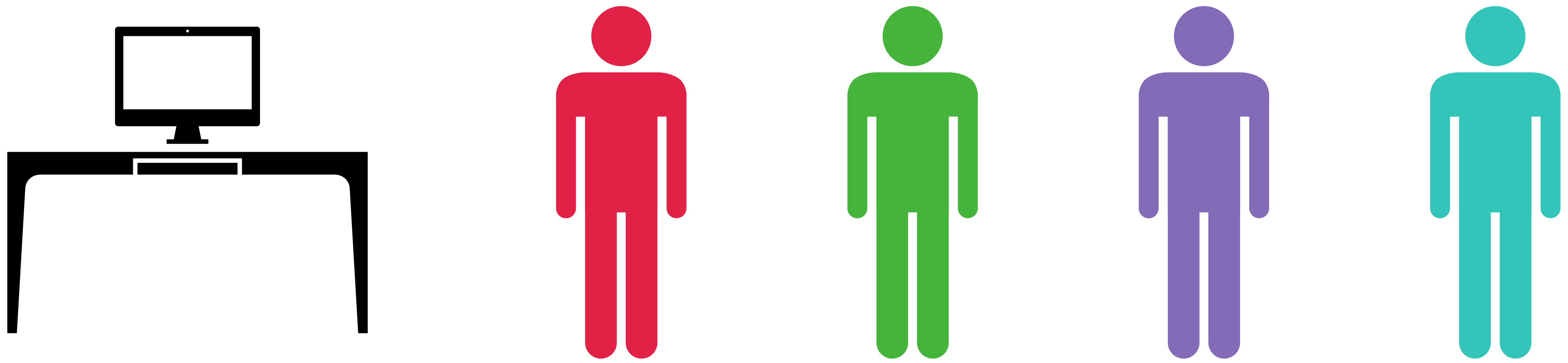

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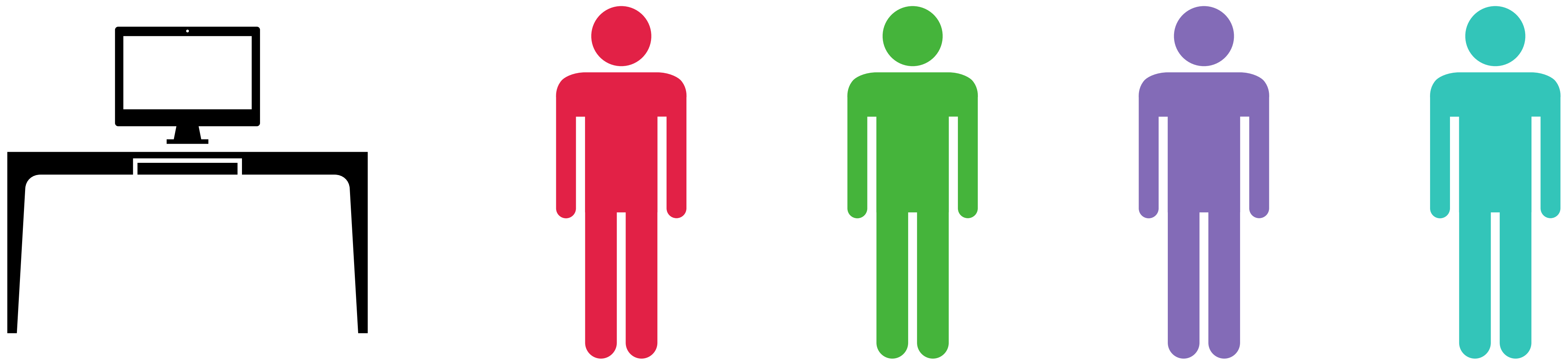

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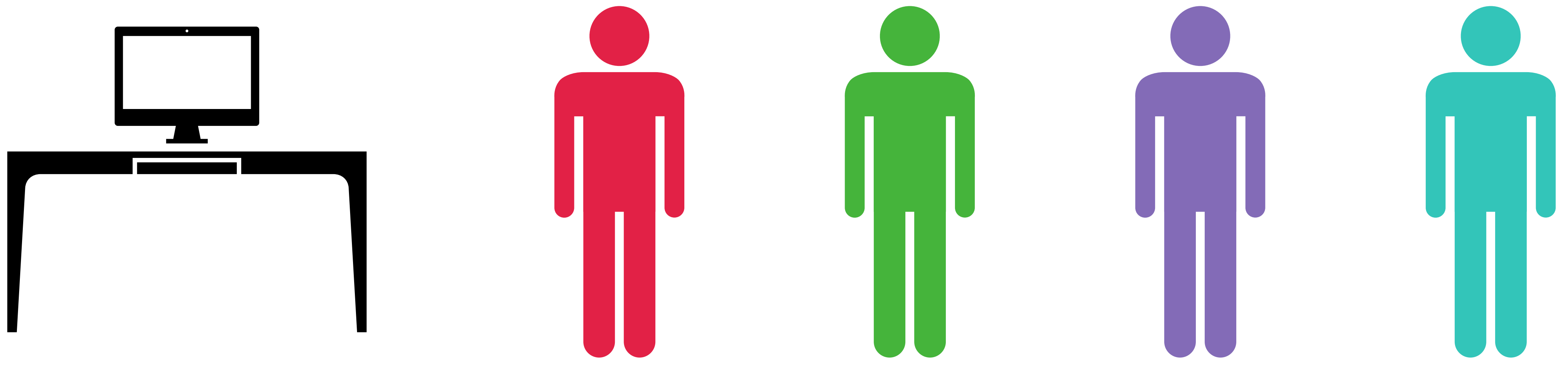

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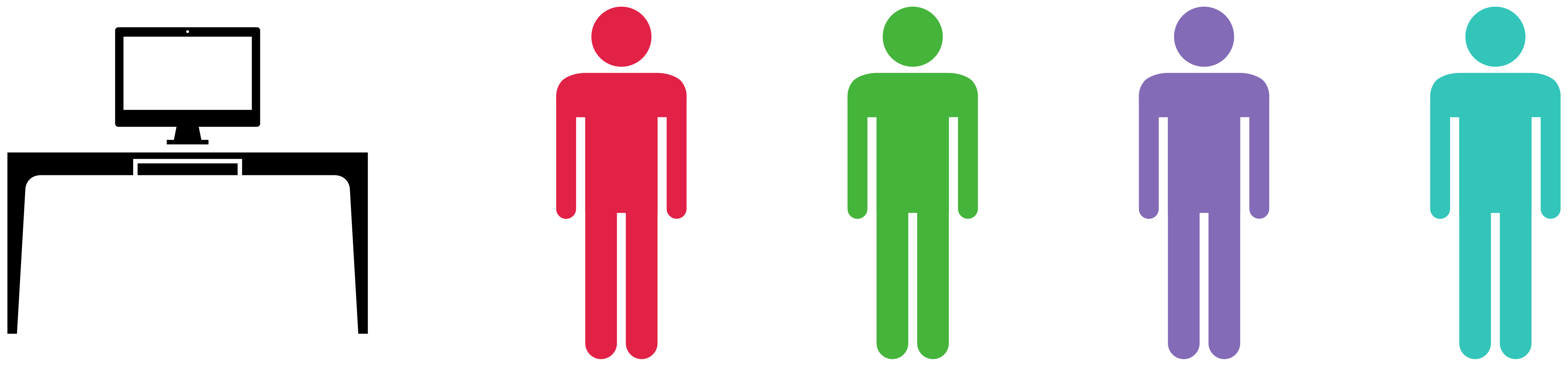
Public University Secretary Problem



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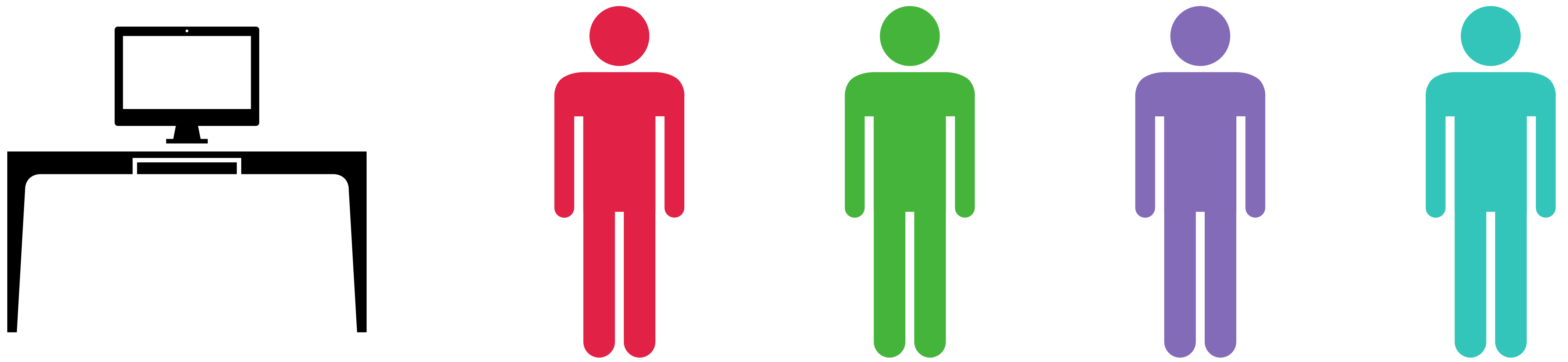

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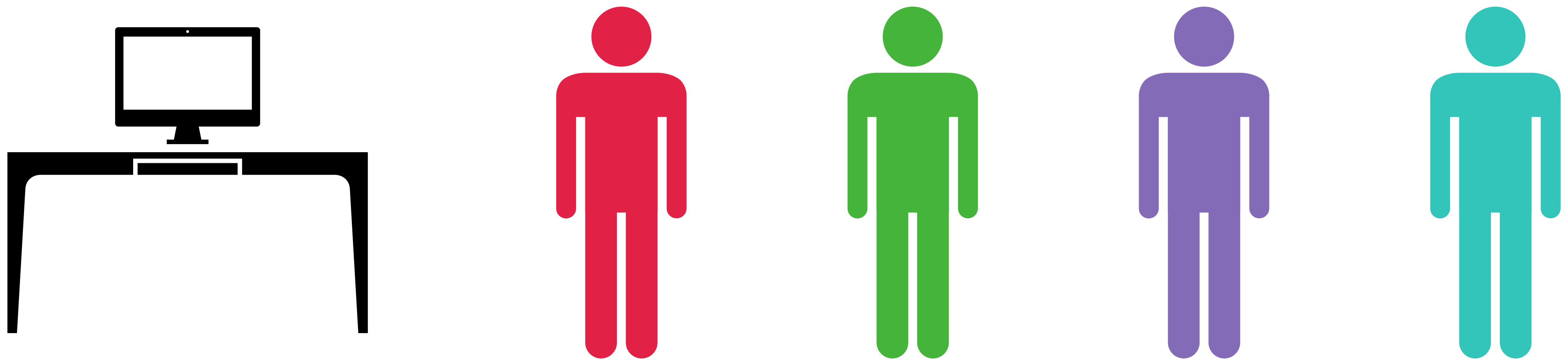

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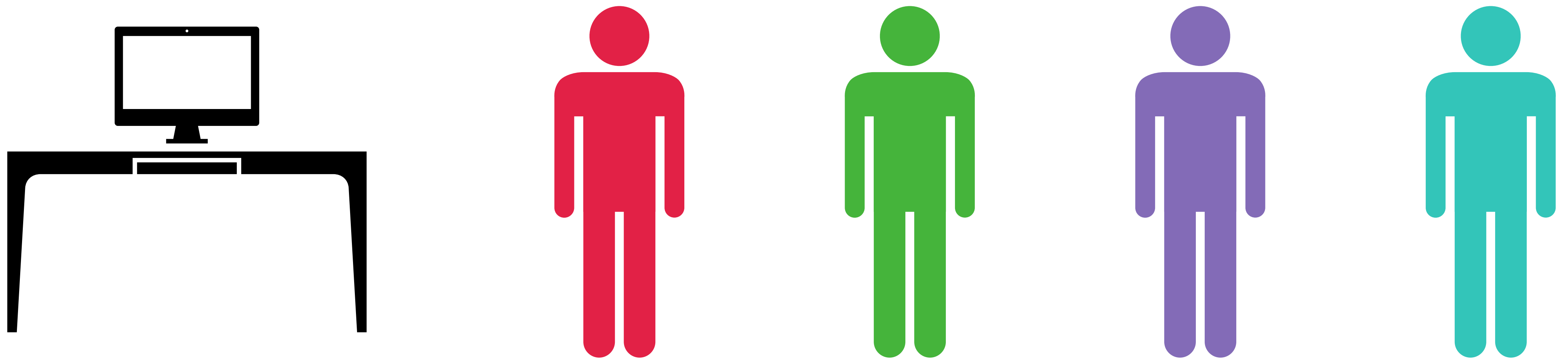

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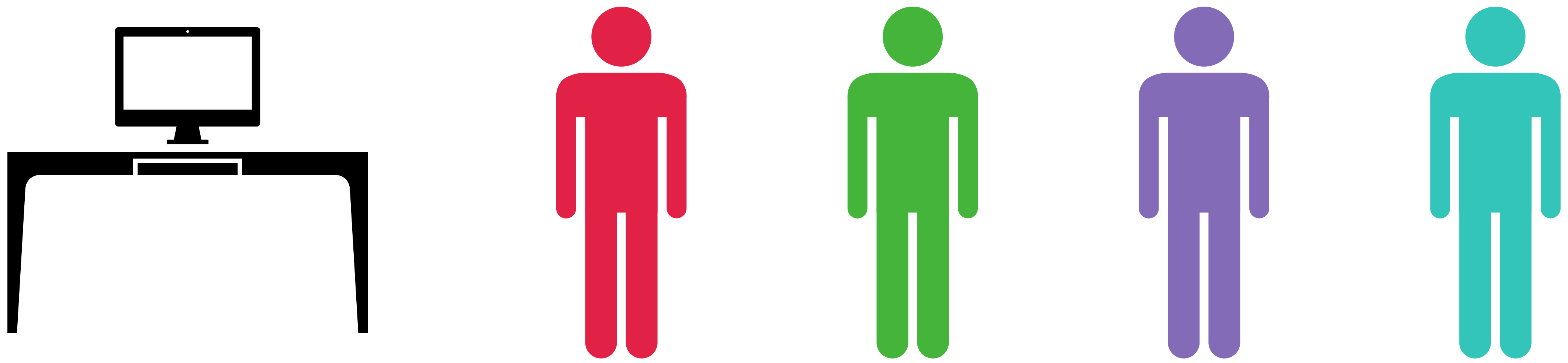

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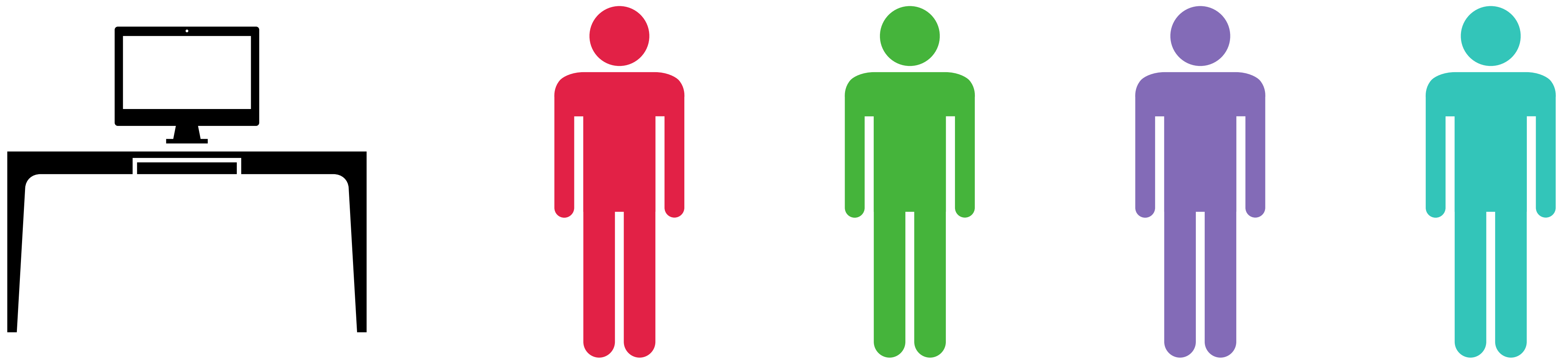

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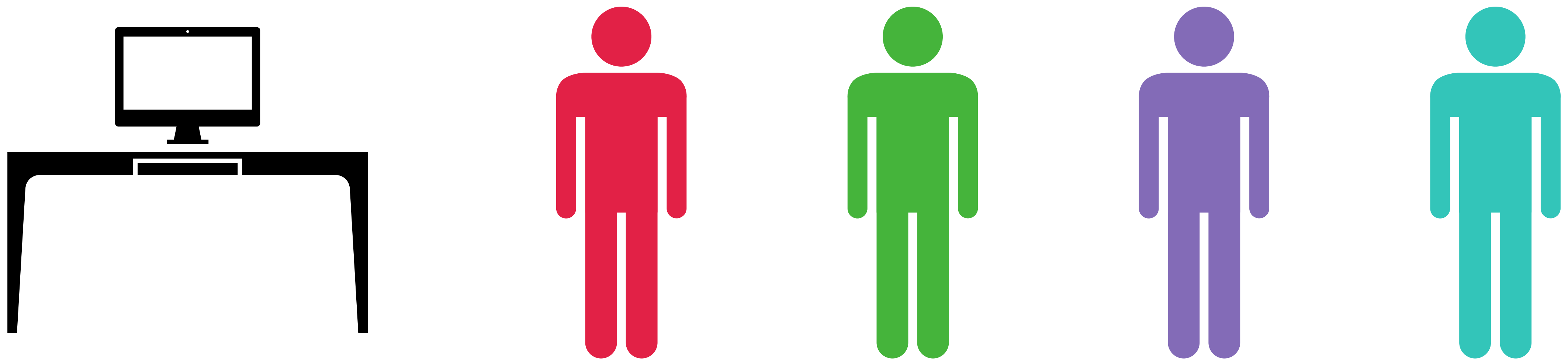

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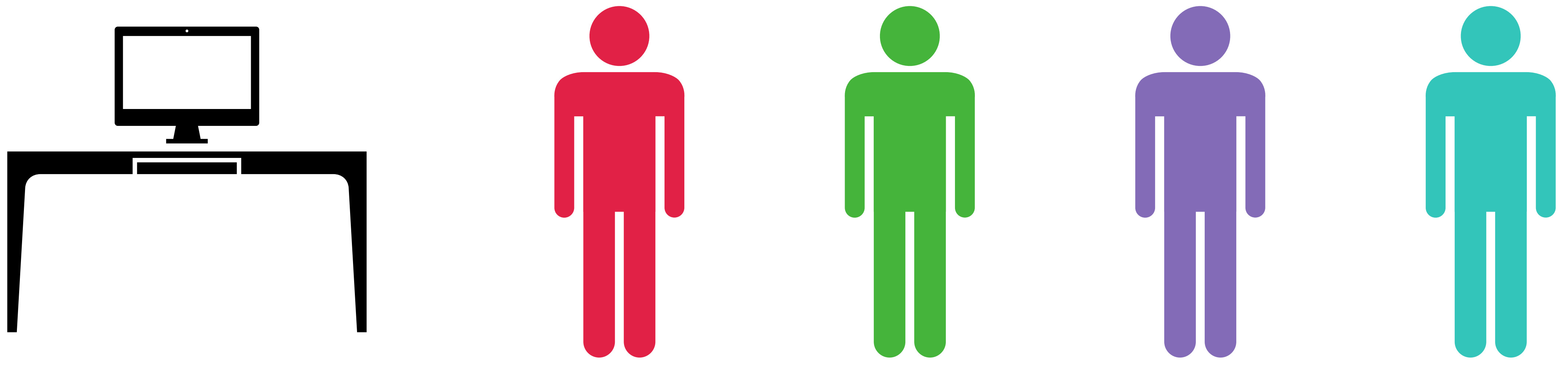

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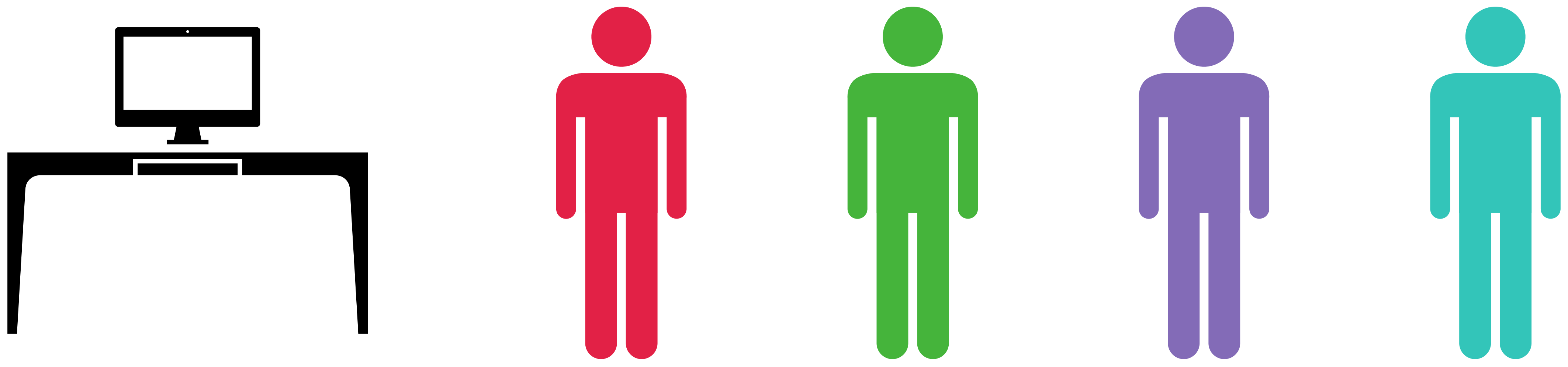

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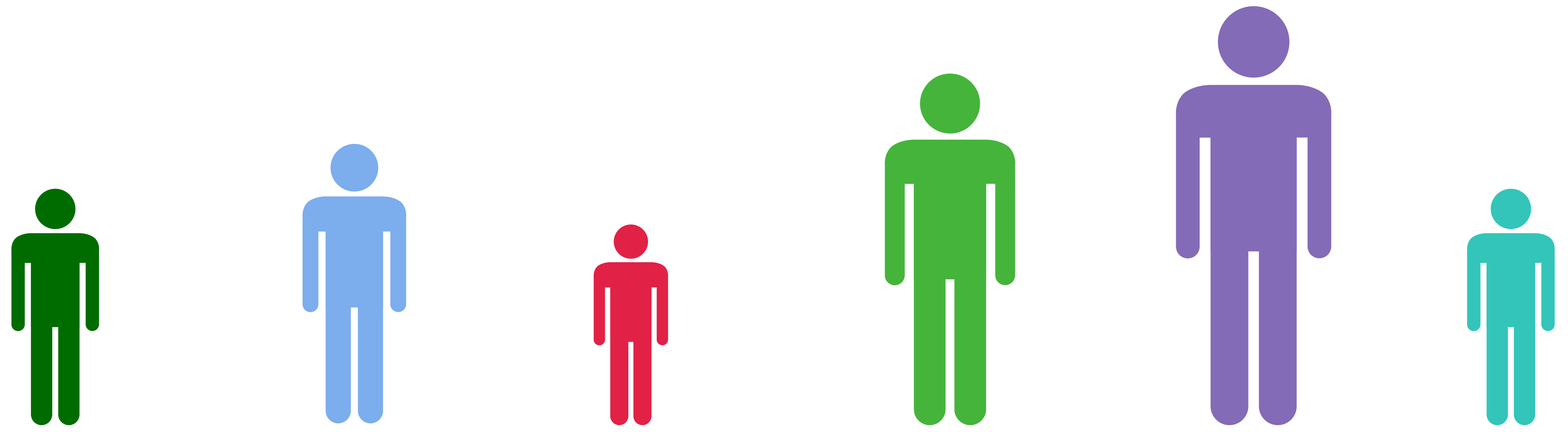
All based on maximization objectives

All require beyond-worst-case approach to learn scale of secretary quality

Sample-and-price for RAND



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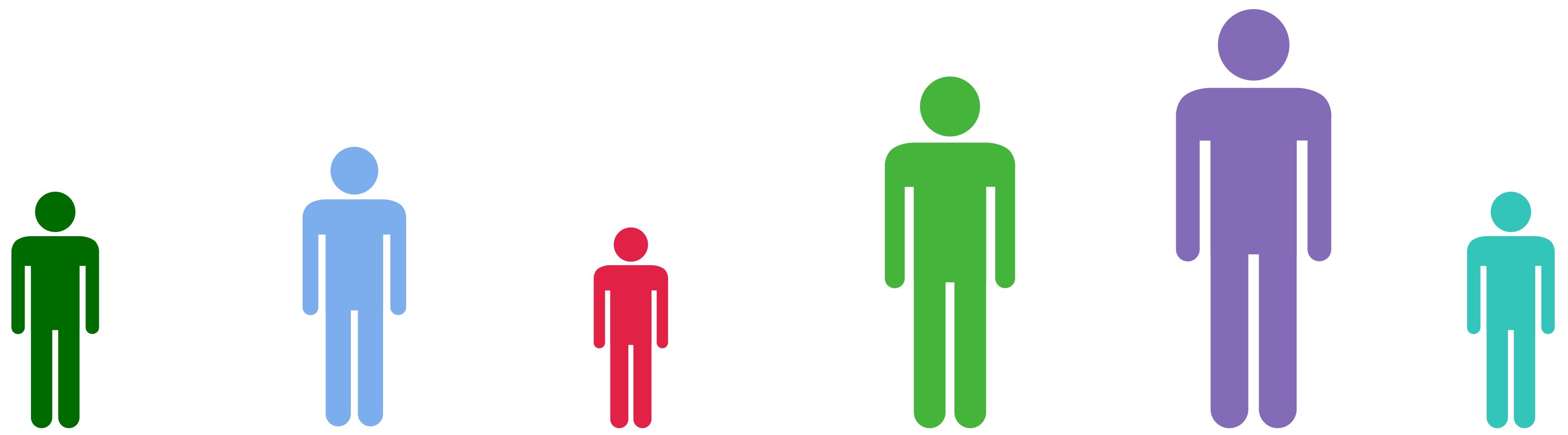

Random Order Arrivals

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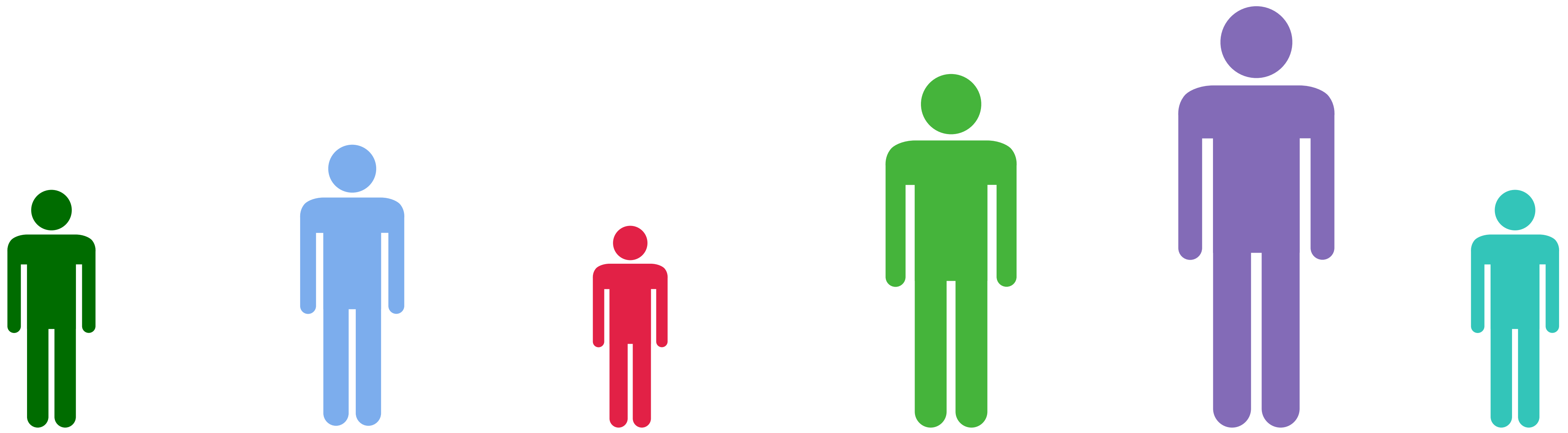

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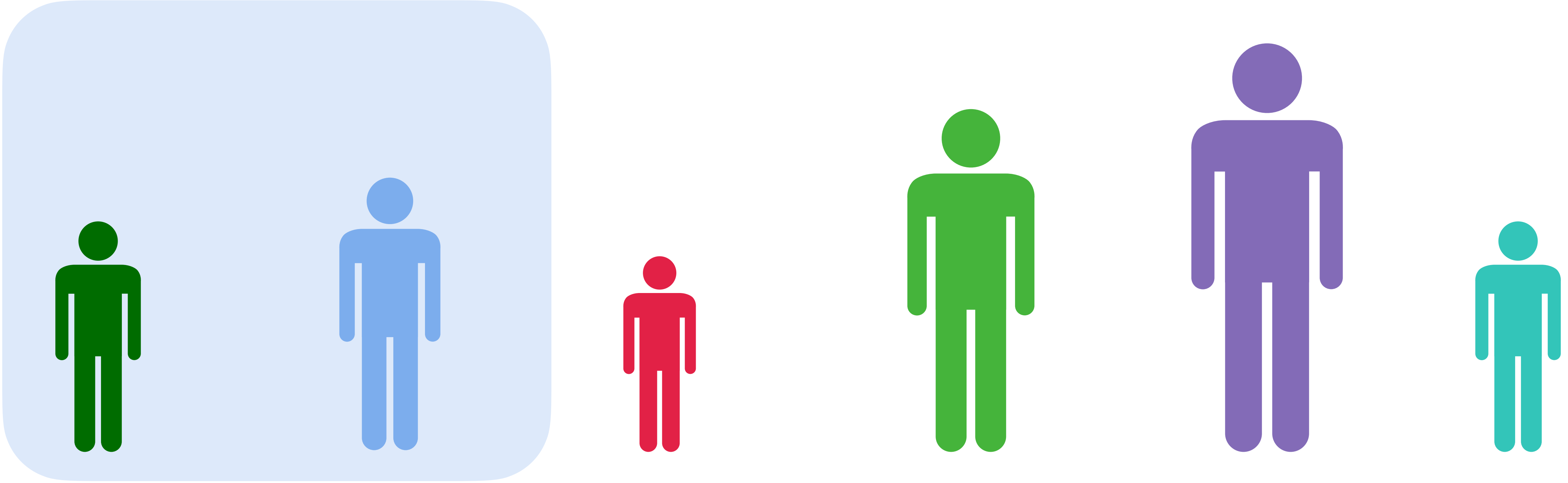

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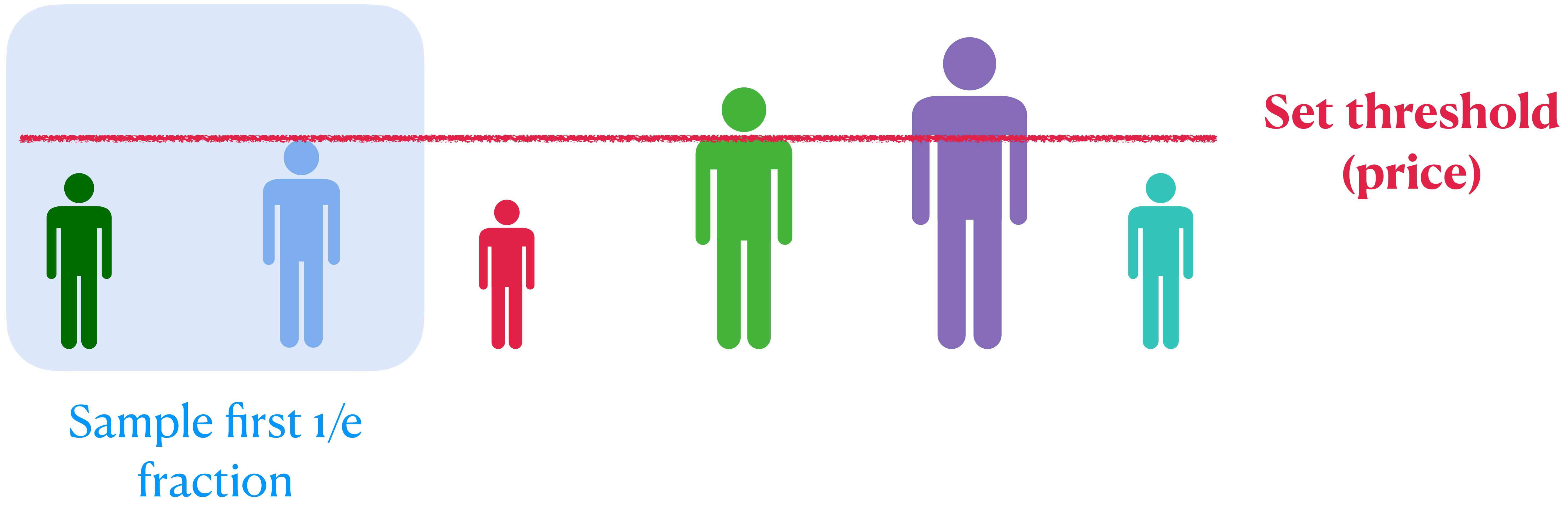

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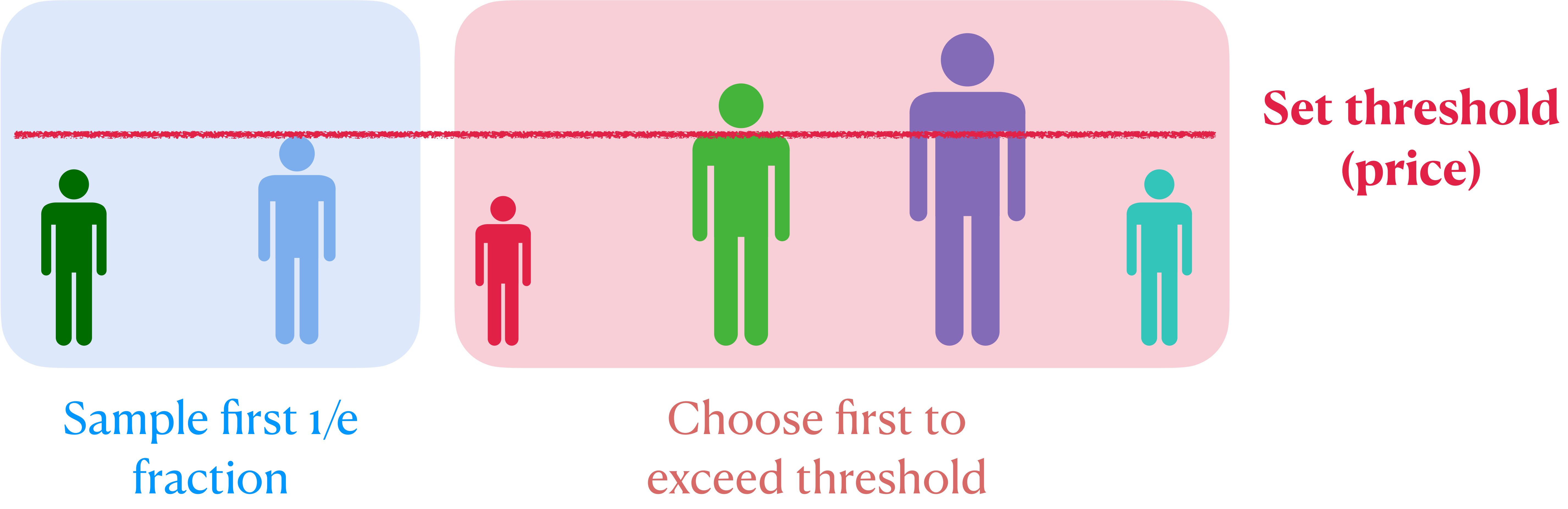

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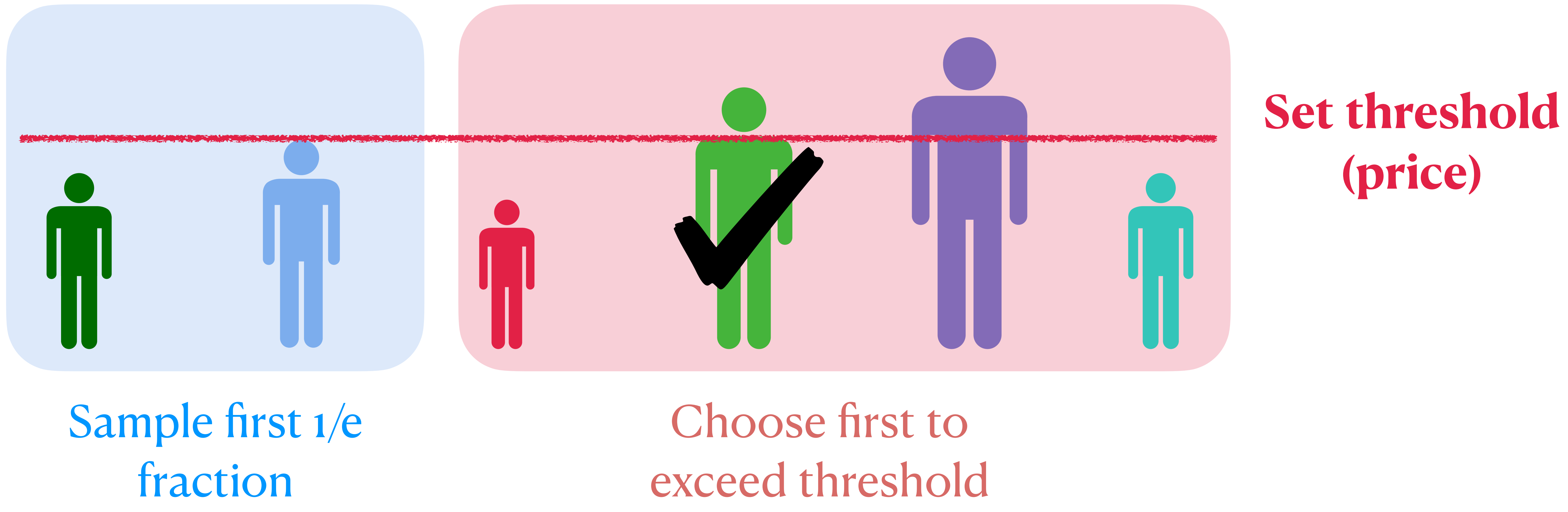

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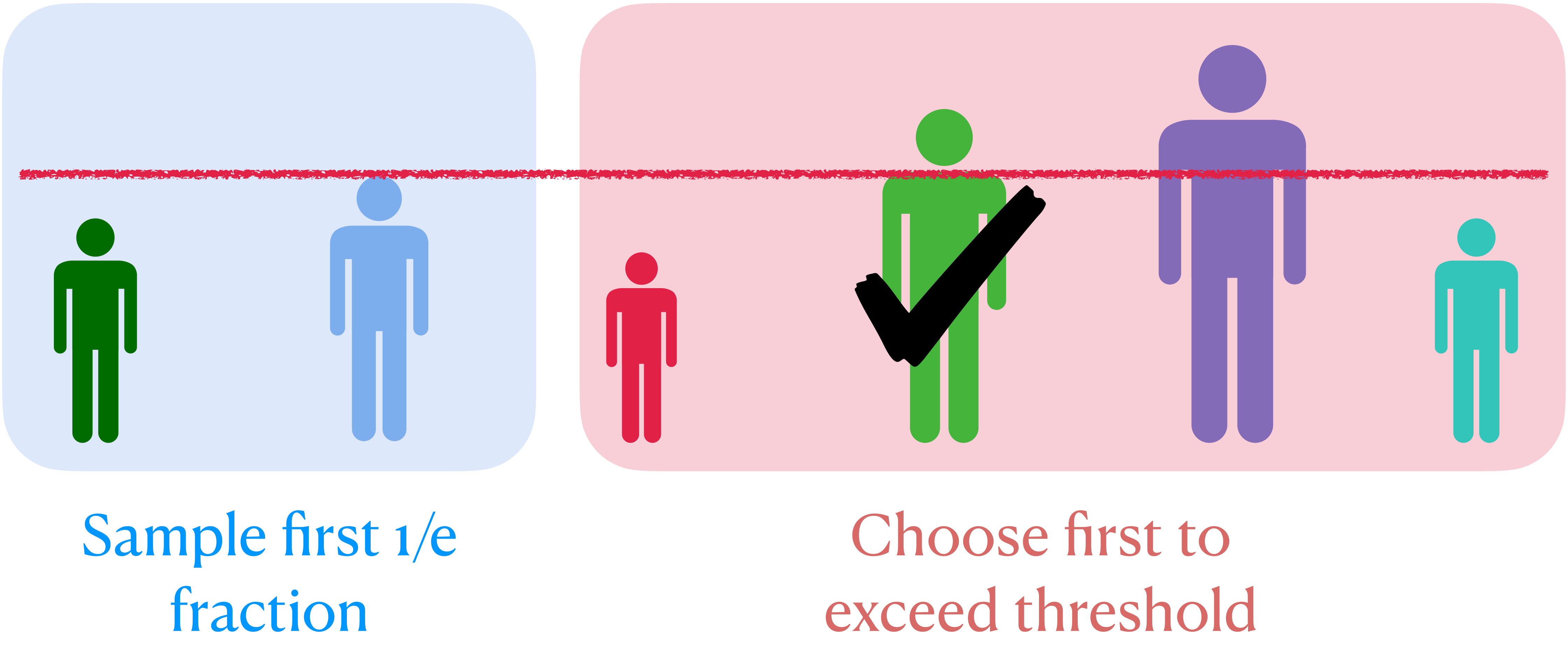


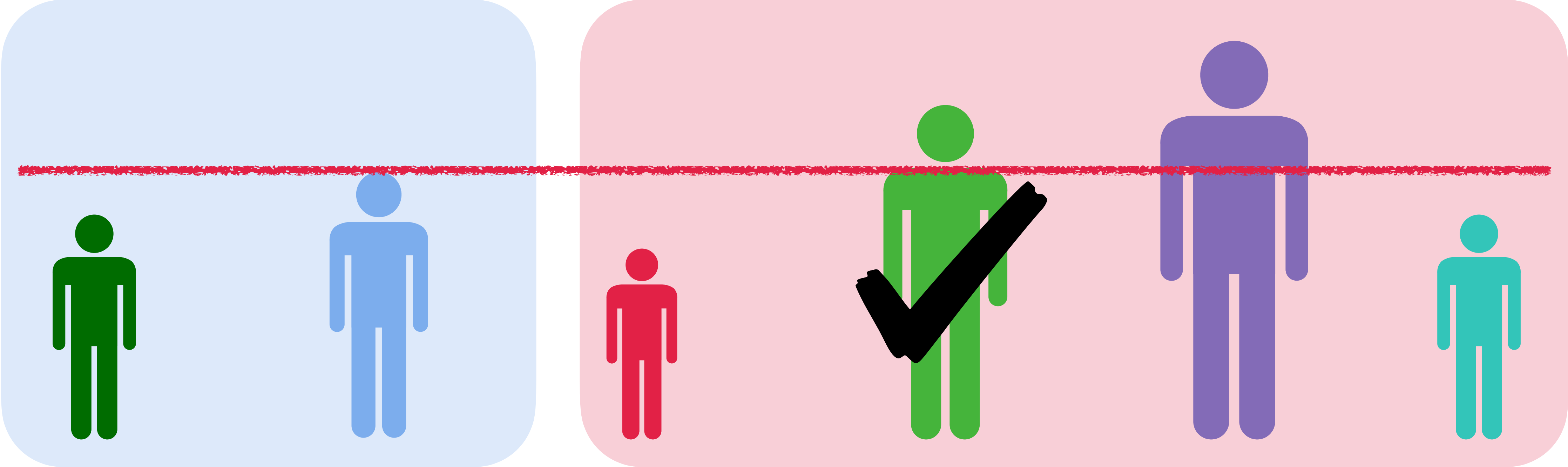
Set threshold (price)

Random Order Arrivals

Know n

Upshot: only need to select most valuable candidate **with *some* (constant) probability**

Sample-and-price for RAND



Sample first 1/e fraction

Choose first to exceed threshold

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Random Order + Sample → Learn **scale** of quality in applicant pool

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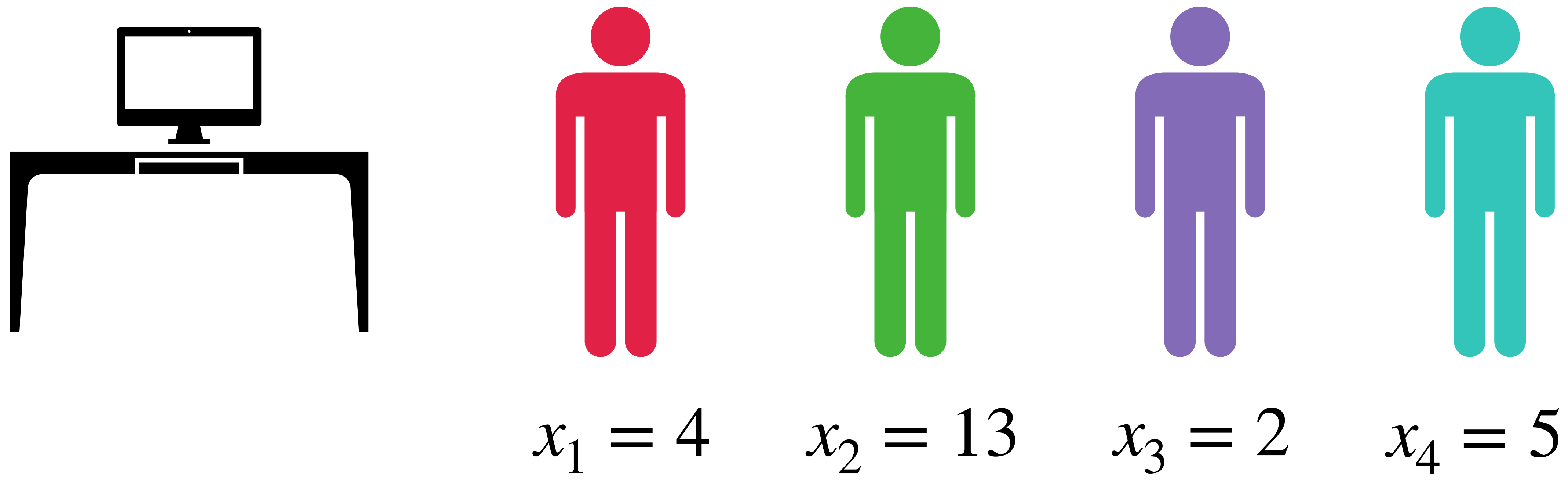

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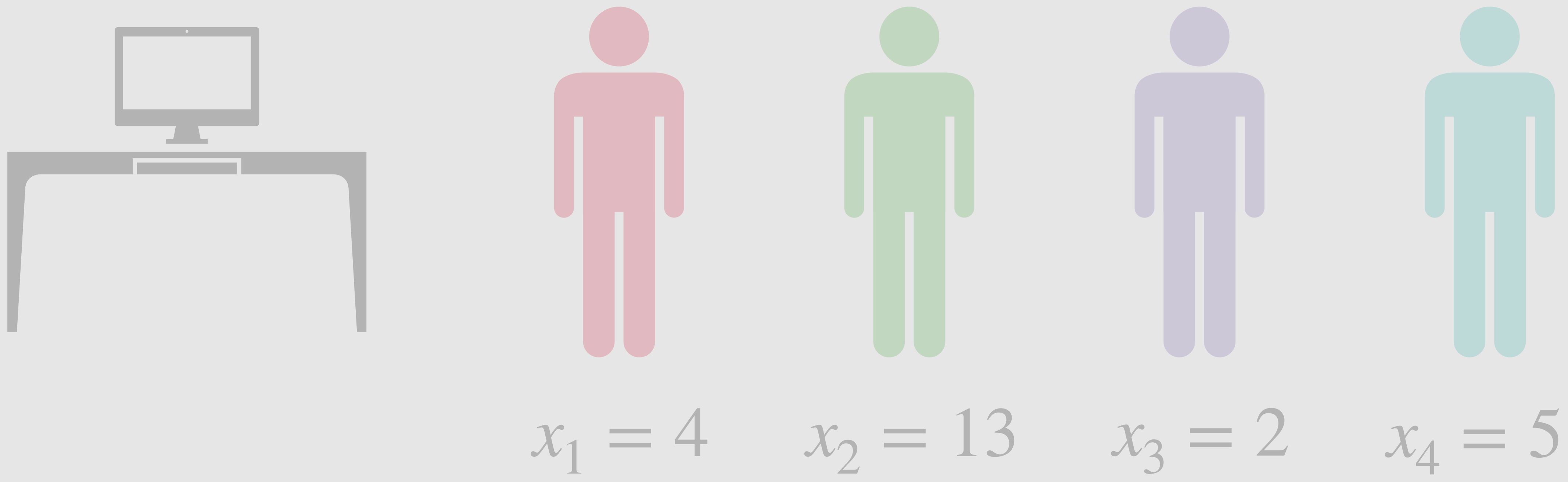

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Random Order

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Public University **Lower Bound**

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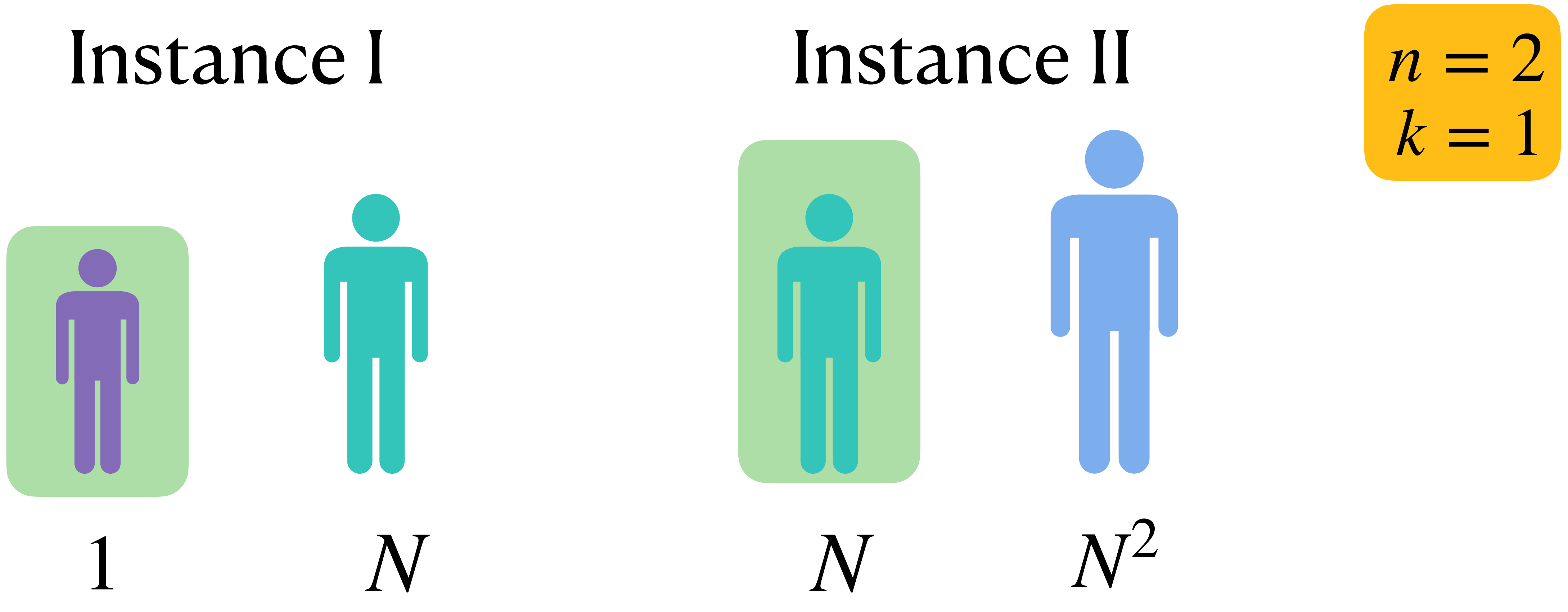

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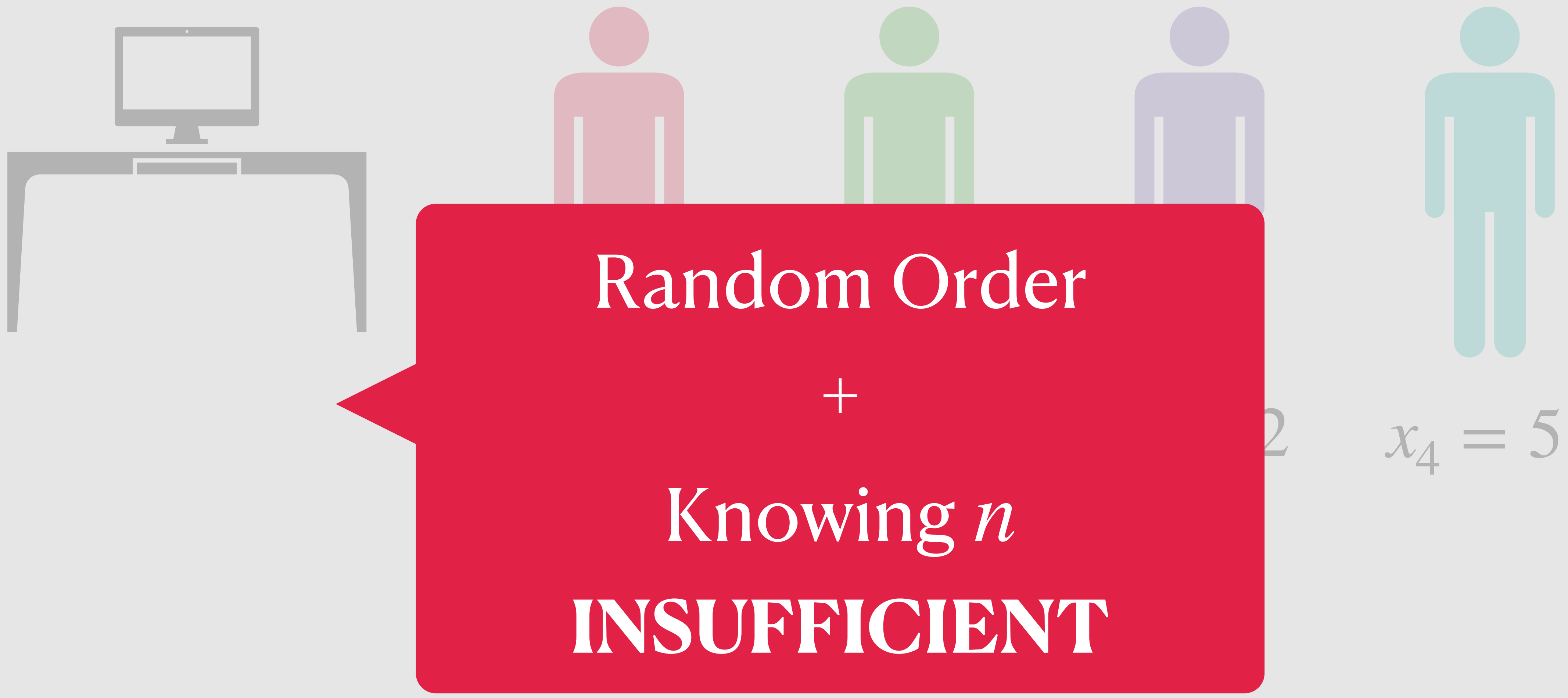

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Instance II

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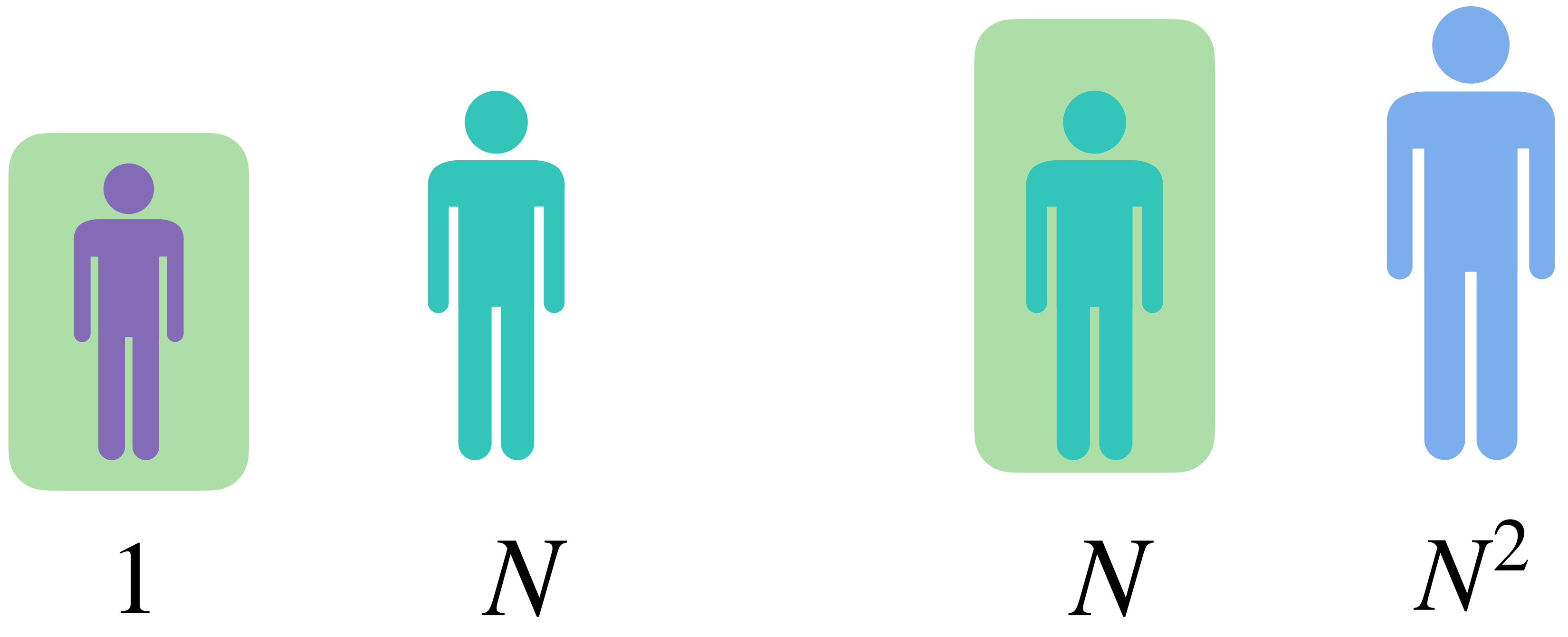

$N \gg 1$

unbounded competitiveness!

Public University Secretary Problem

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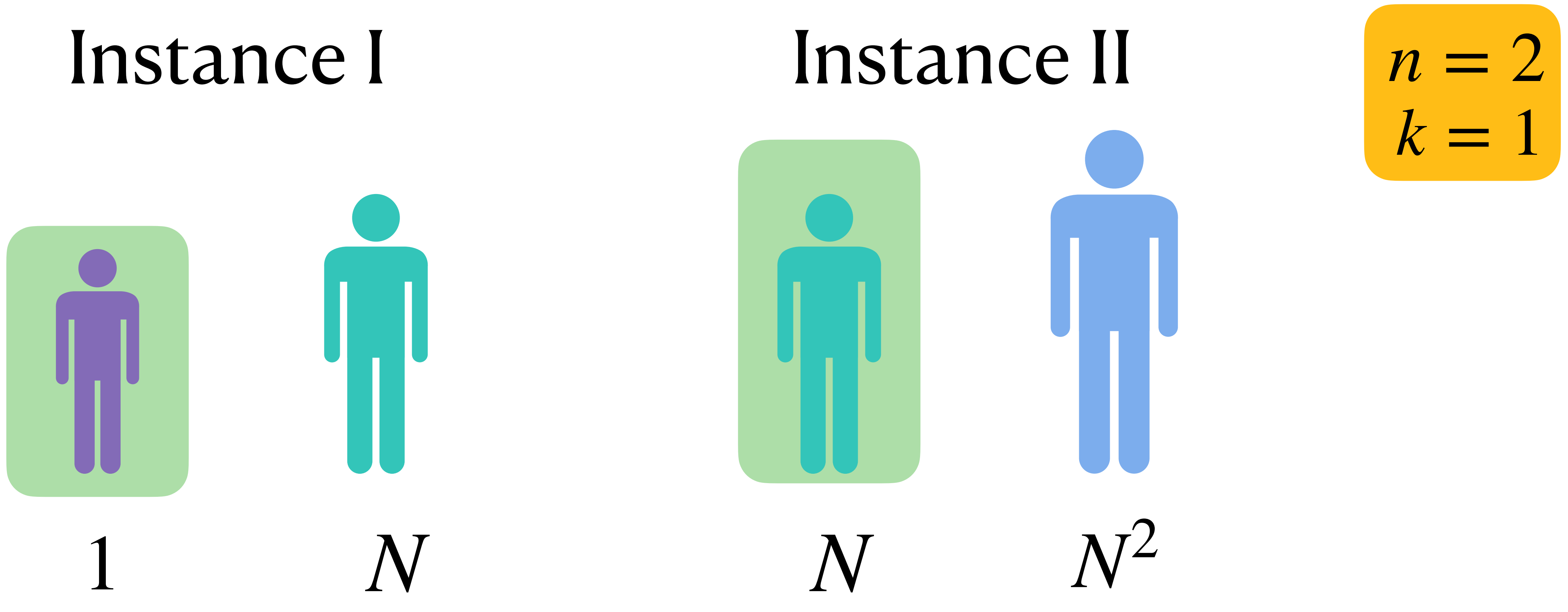

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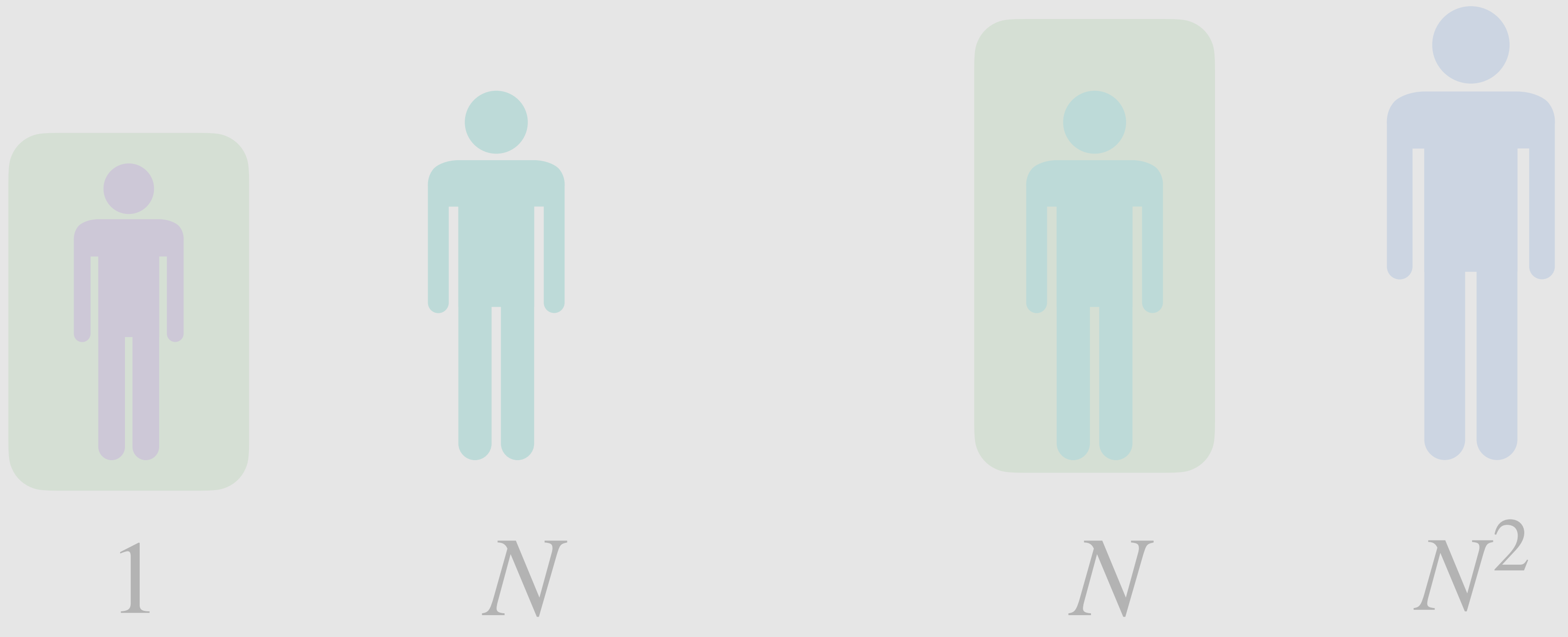

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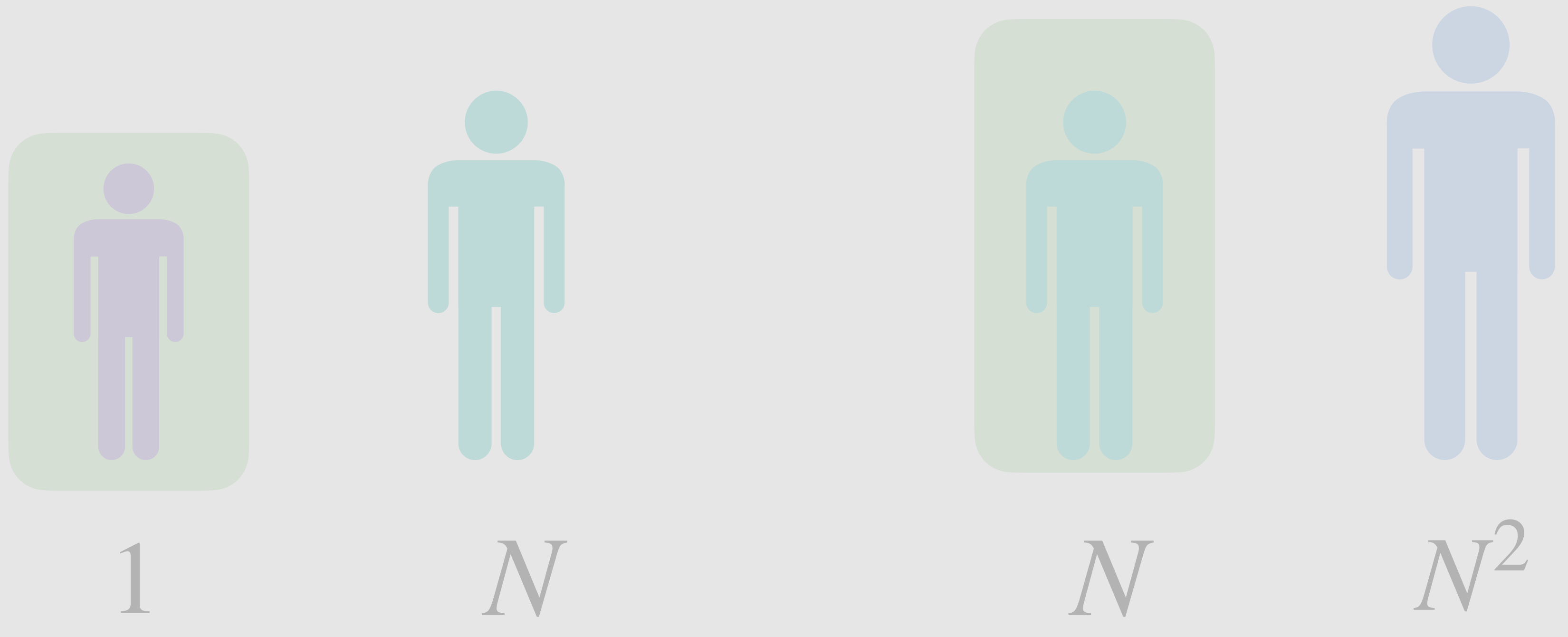

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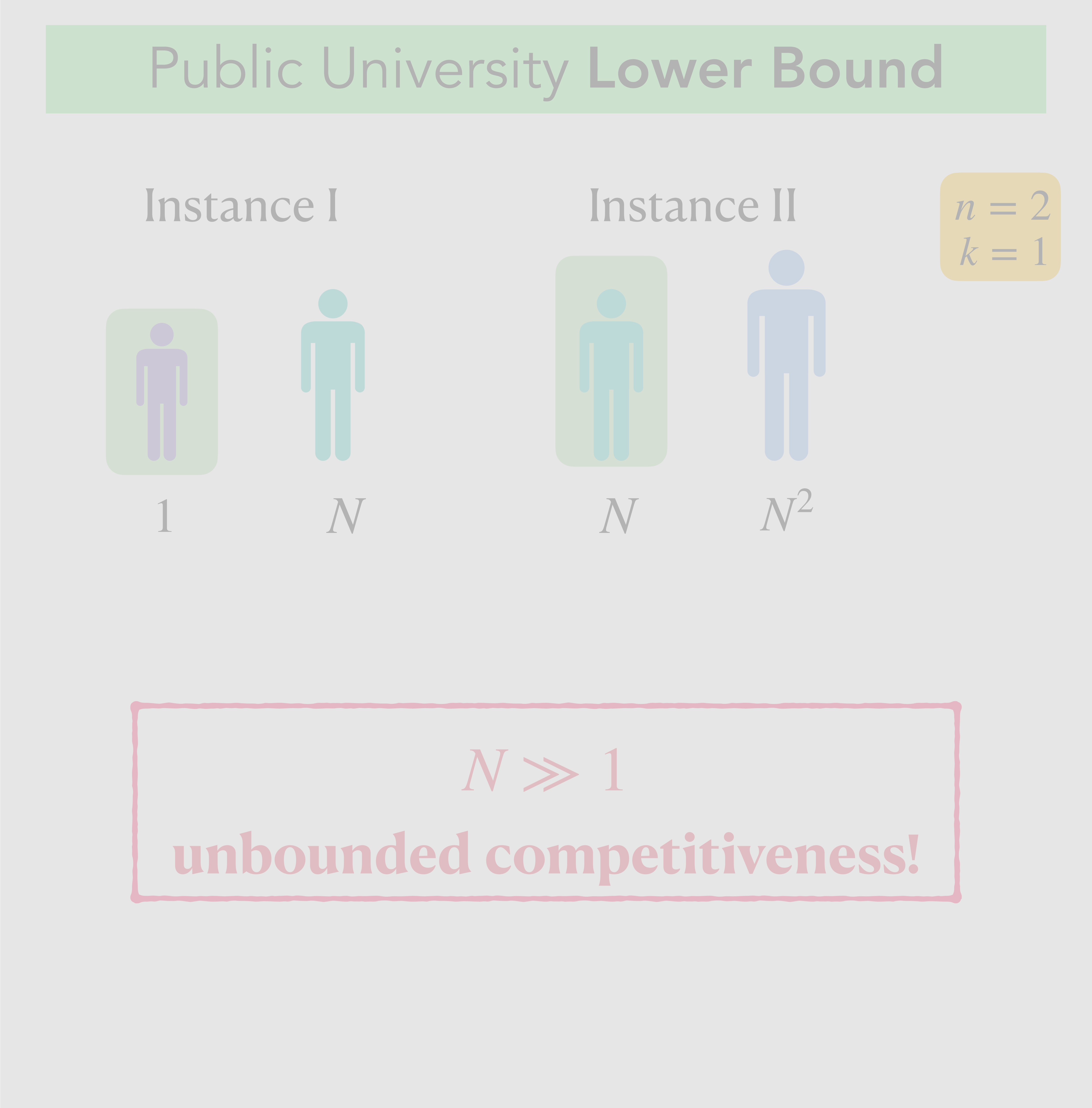

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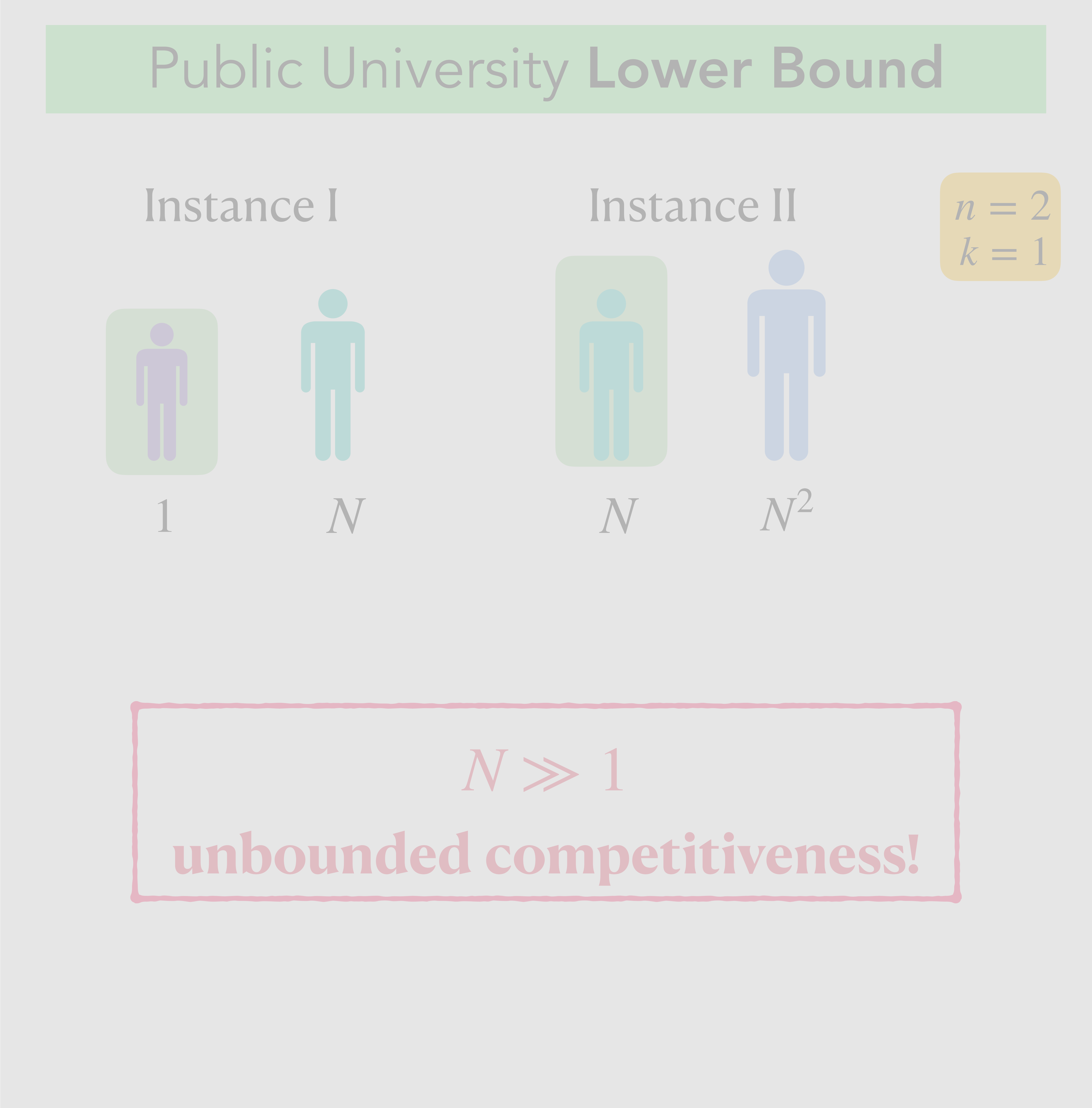

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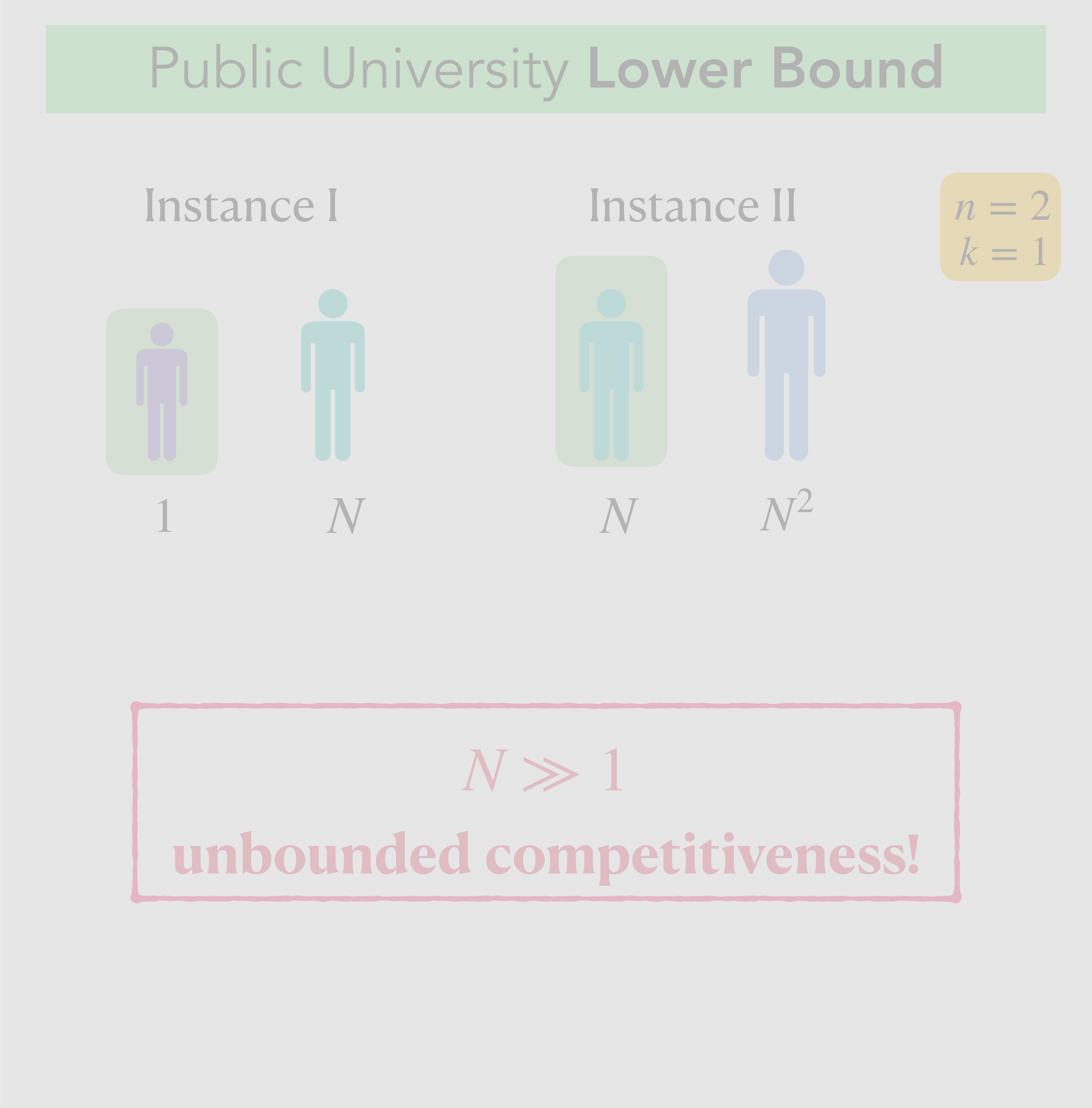
- Public University is a **minimization problem**
- Cannot simply reject secretaries



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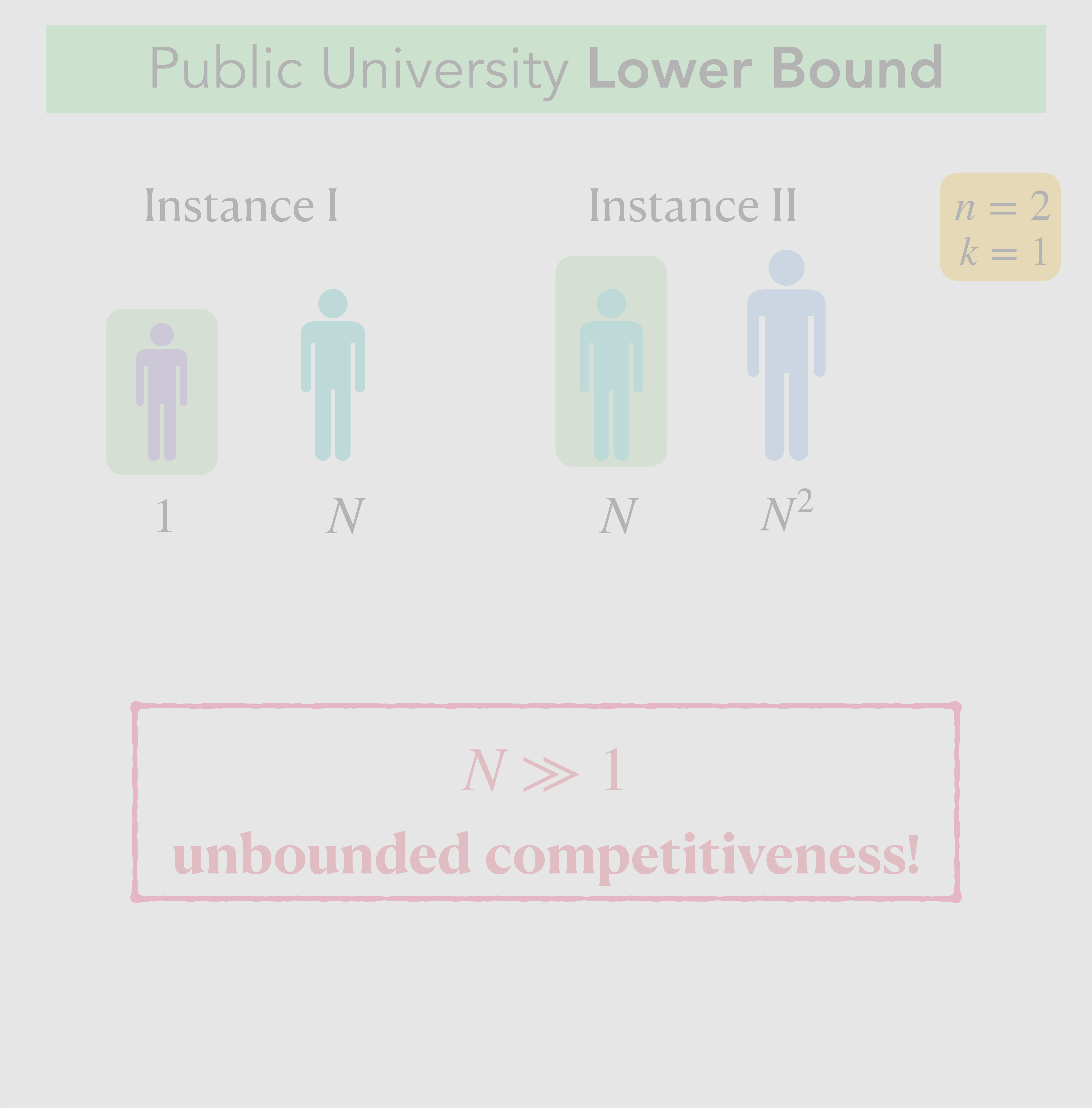
- Public University is a **minimization problem**
- Cannot simply reject secretaries
  - **Must** hire **at least** k secretaries



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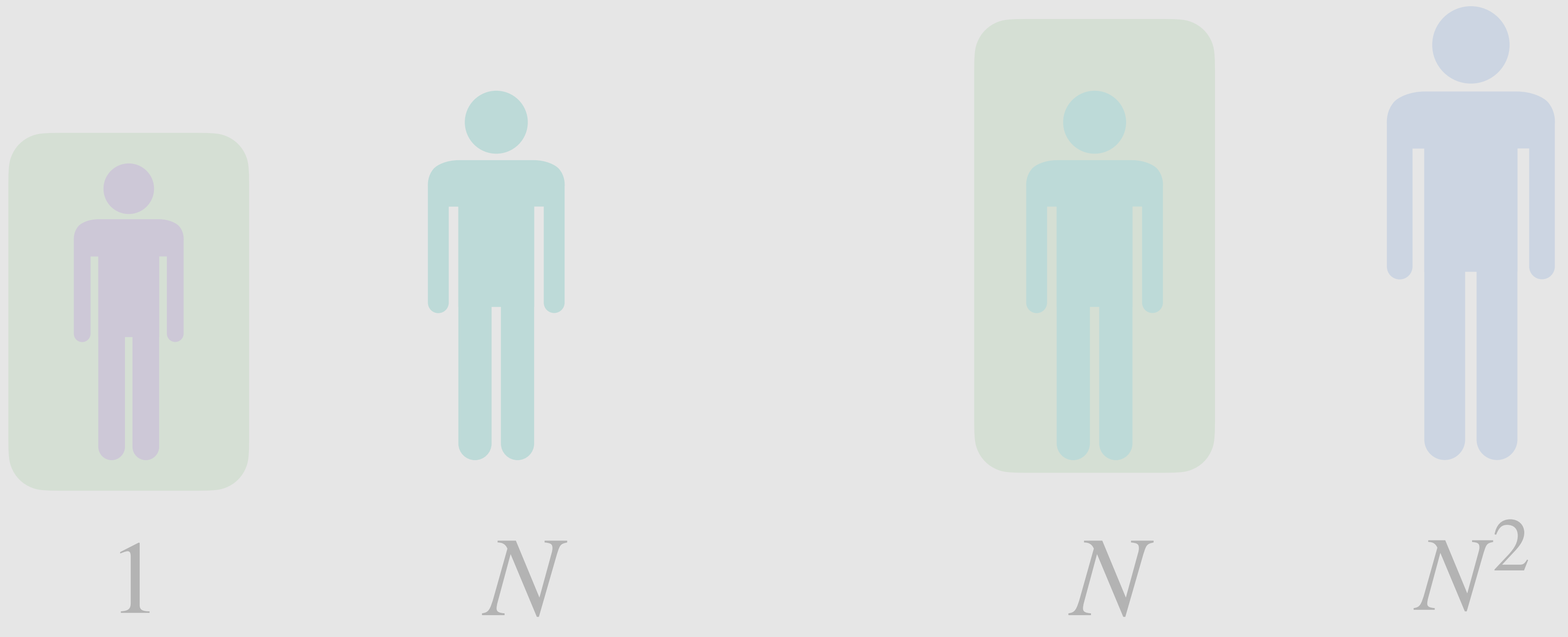
  - Could be **forced to incur enormous cost**

Public University **Lower Bound**

Instance I | Instance II

$n = 2$
$k = 1$



$N \gg 1$

unbounded competitiveness!

Why is random order not sufficient?

- Public University is a **minimization problem**
- Cannot simply reject secretaries
  - **Must** hire **at least** k secretaries
  - Could be **forced to incur enormous cost**
- RAND / maximization: ignore cases where low-value secretaries hired

How do we break through the strong lower bound?

How do we break through the strong lower bound?

Learning-augmented approach:

How do we break through the strong lower bound?

Learning-augmented approach:

Online algorithm given "budget" B **a priori**

B **upper bound on OPT** (cost of k cheapest secretaries)

Results

Results

Best possible online algorithm for Public University Secretary

$\Theta(\log k)$-competitive against B

in both **adversarial and random** arrival orders

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Upper Bound of $O(B \cdot \log k)$:

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Key Takeaway: randomization of negligible benefit!

Results

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"Cautious" Algorithm

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"Cautious" Algorithm

(Roughly) *hire candidate i iff they are in "best" solution up until now*

Lower Bound of $\Omega(B \cdot \log k)$:

✓ Even with **random** ordering

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Results

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"Cautious" Algorithm

(Roughly) *hire candidate i iff they are in "best" solution up until now*

Lower Bound of $\Omega(B \cdot \log k)$:

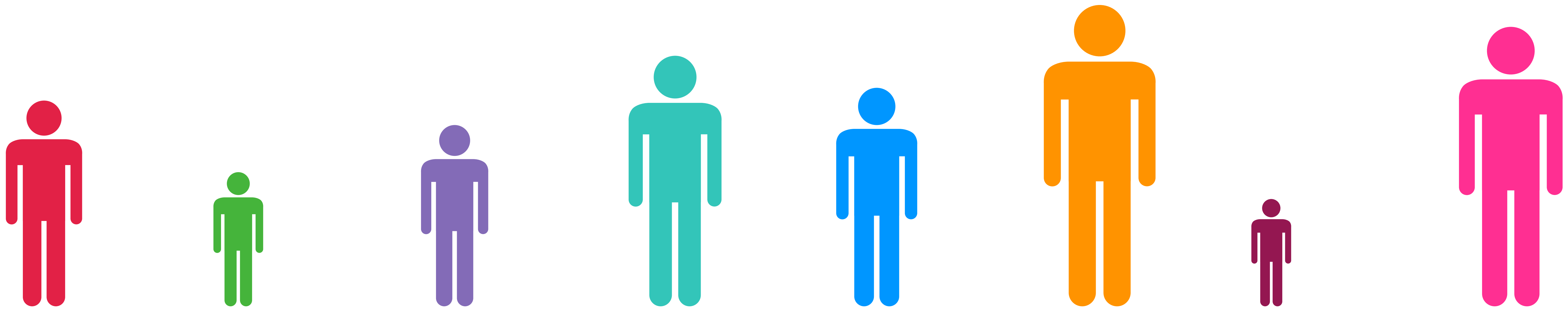
✓ Even with **random** ordering

✓ Against **randomized** algorithms

Key step: (roughly) any competitive algorithm must hire each candidate hired by the cautious algorithm

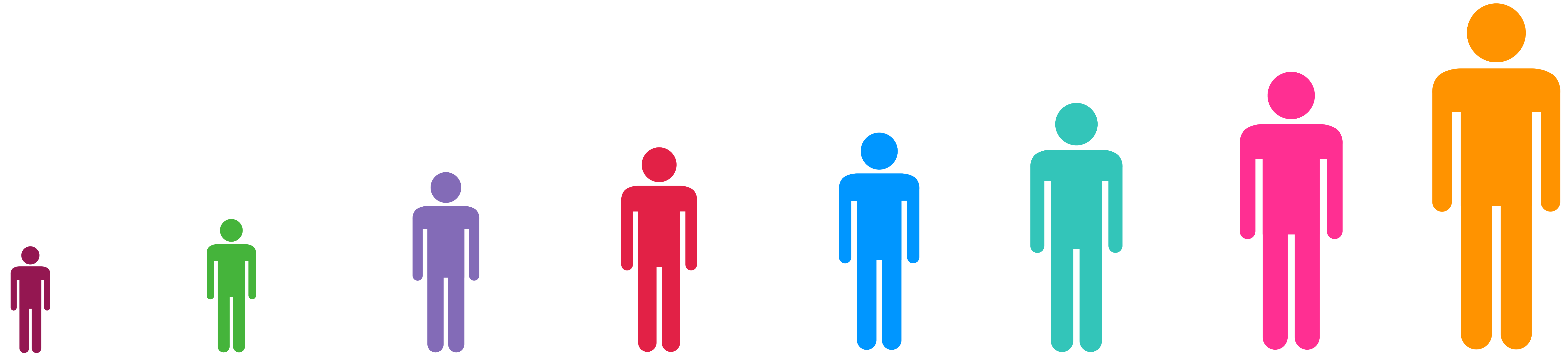
Lower Bound: Adversarial Order

First i candidates: ***(adversarial) arrival order***



Lower Bound: Adversarial Order

First i candidates: ***sorted***



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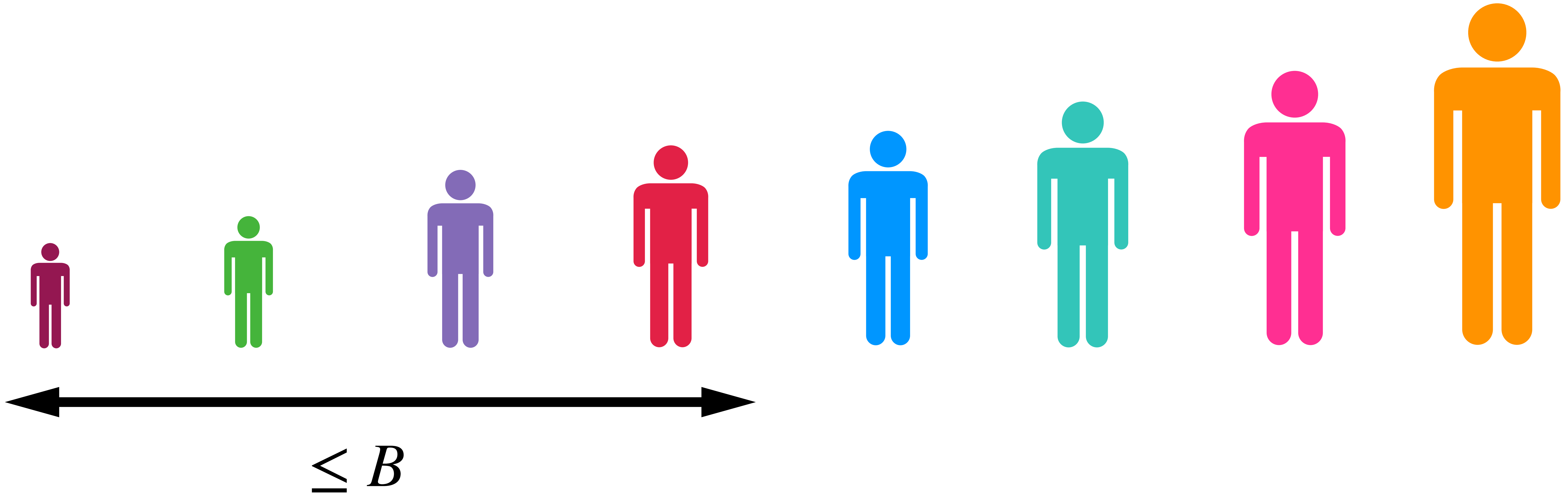

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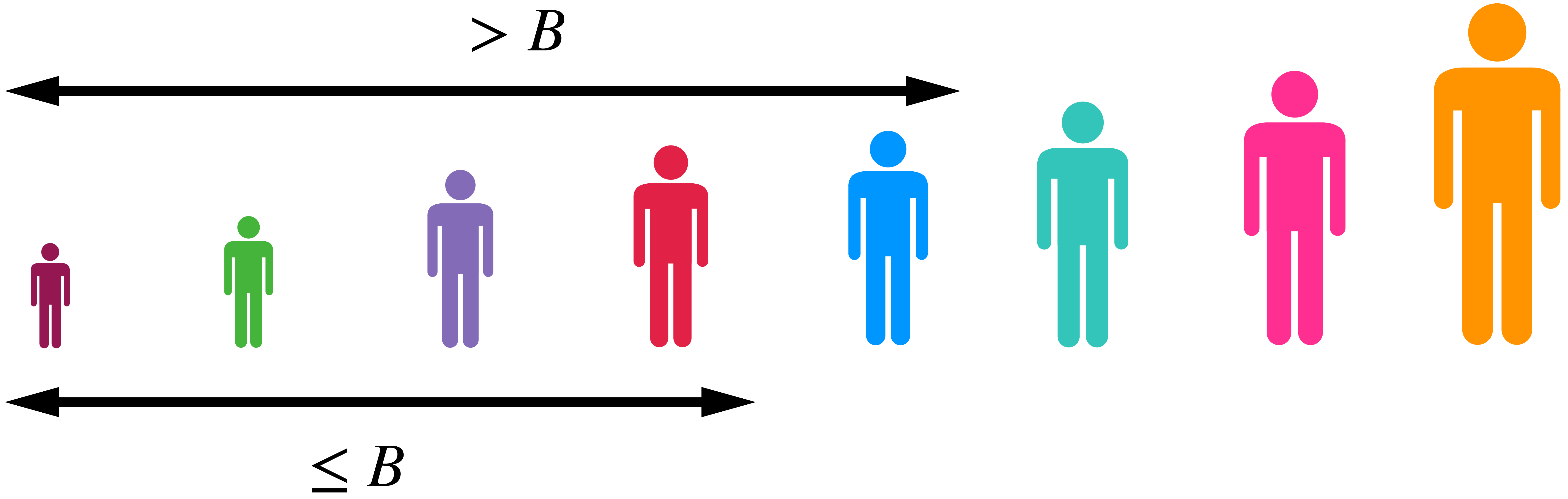

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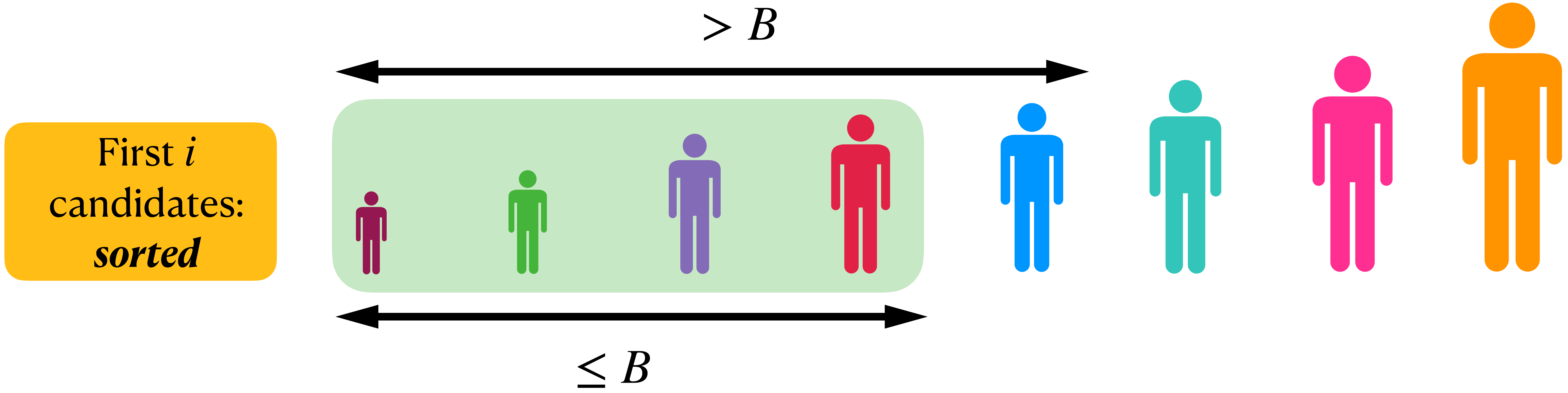

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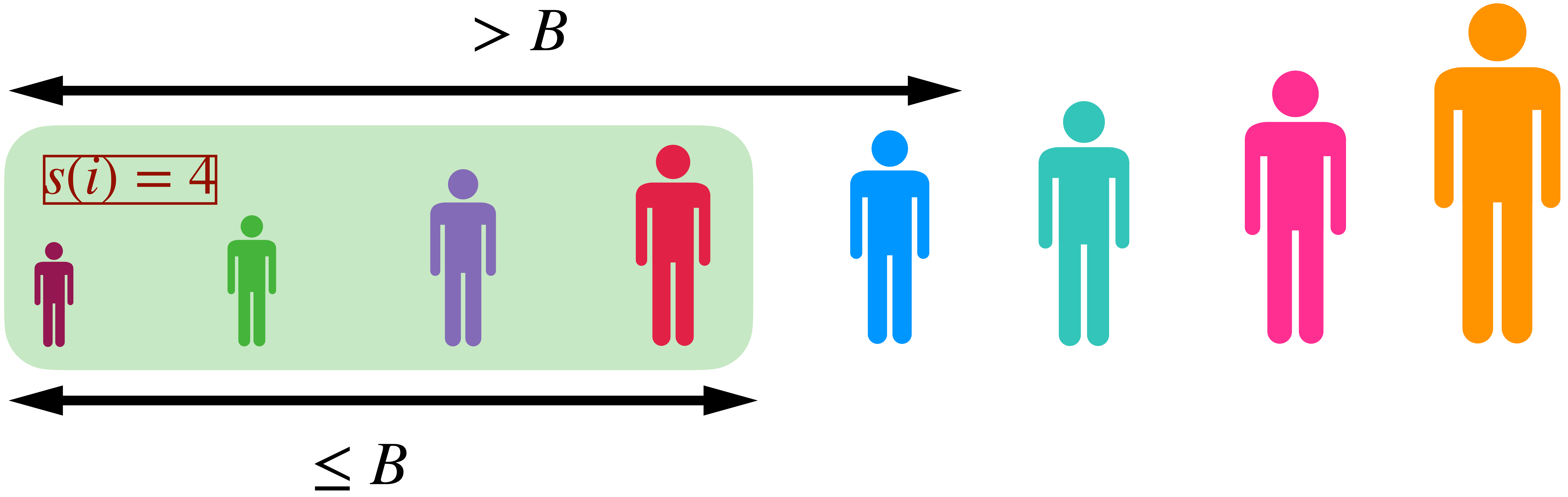

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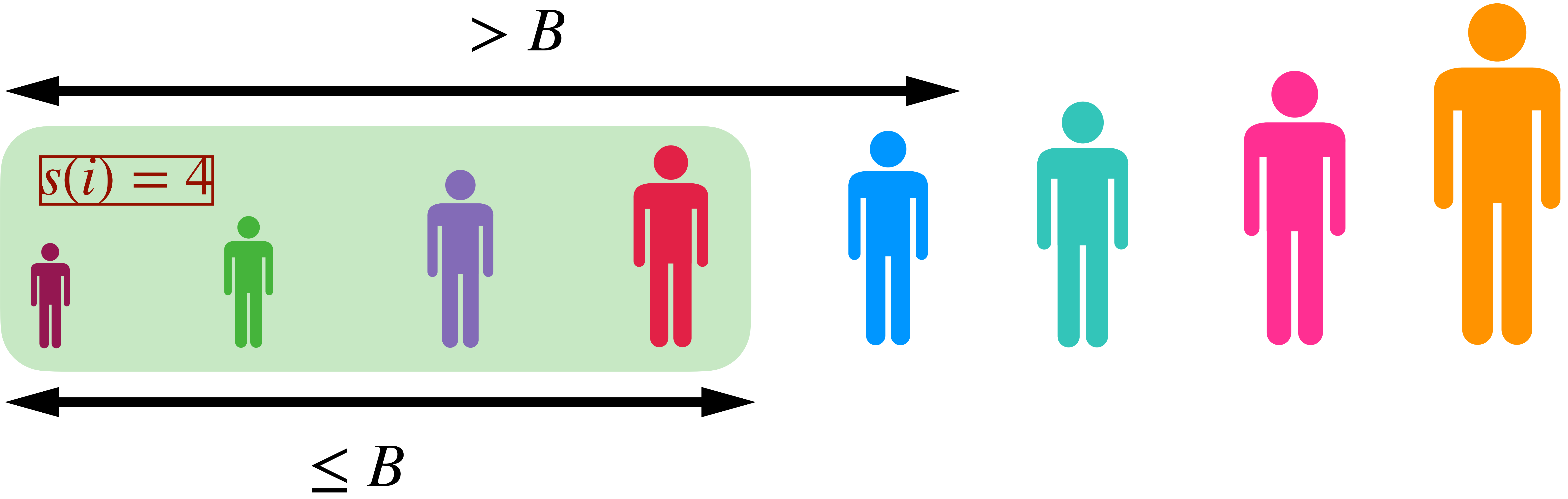
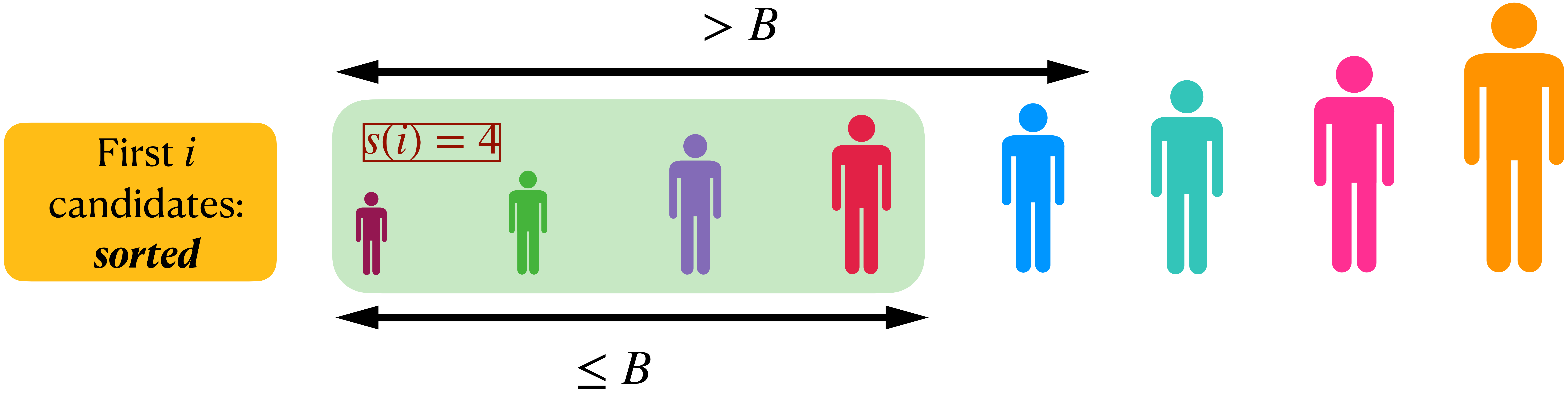


Acceptance Property: with probability 1, hire $s(i)$ candidates among first i candidates, for all i

Lower Bound: Adversarial Order



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Lemma: Every randomized algorithm that is competitive in adversarial order model has the **acceptance property.**

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Idea: high cost secretaries arrive first

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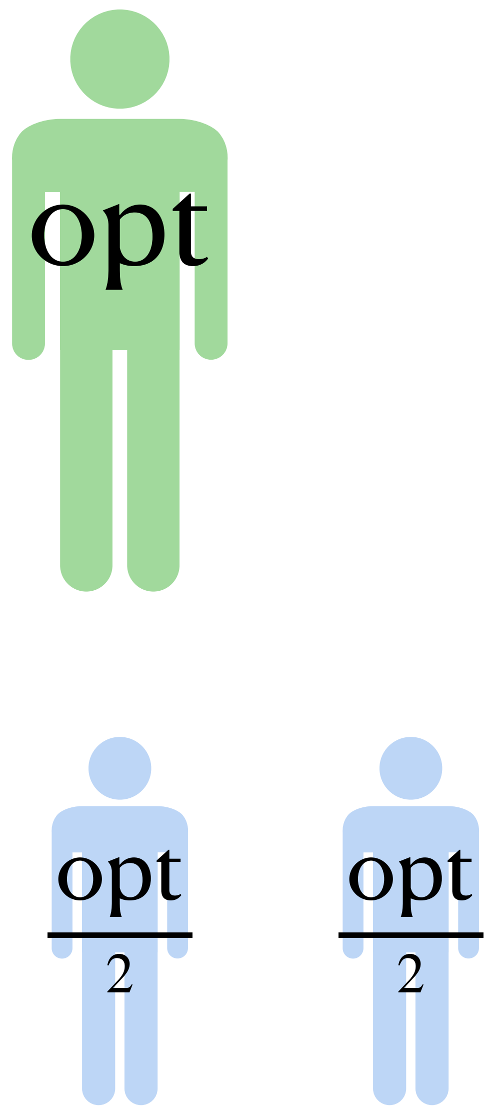

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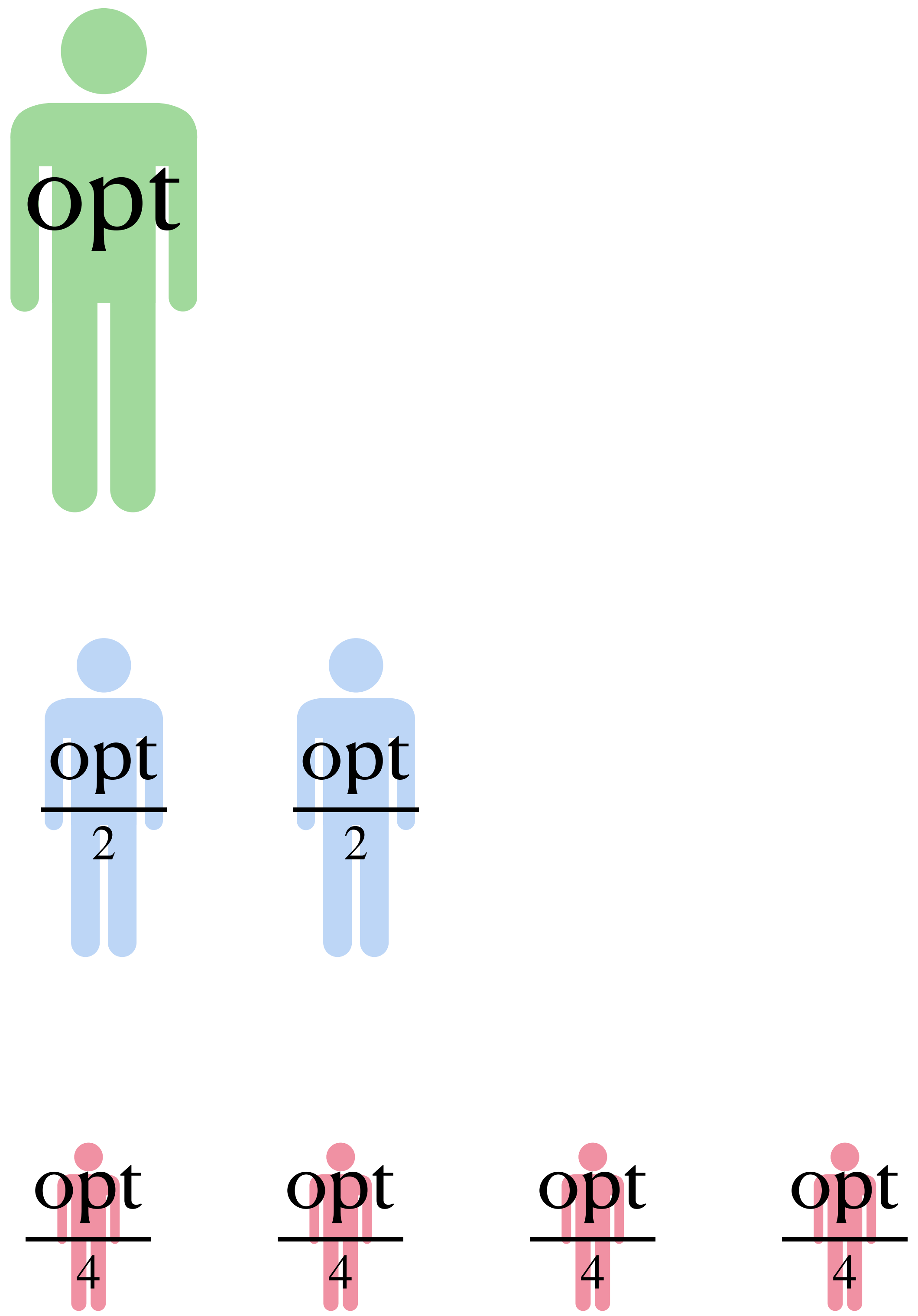

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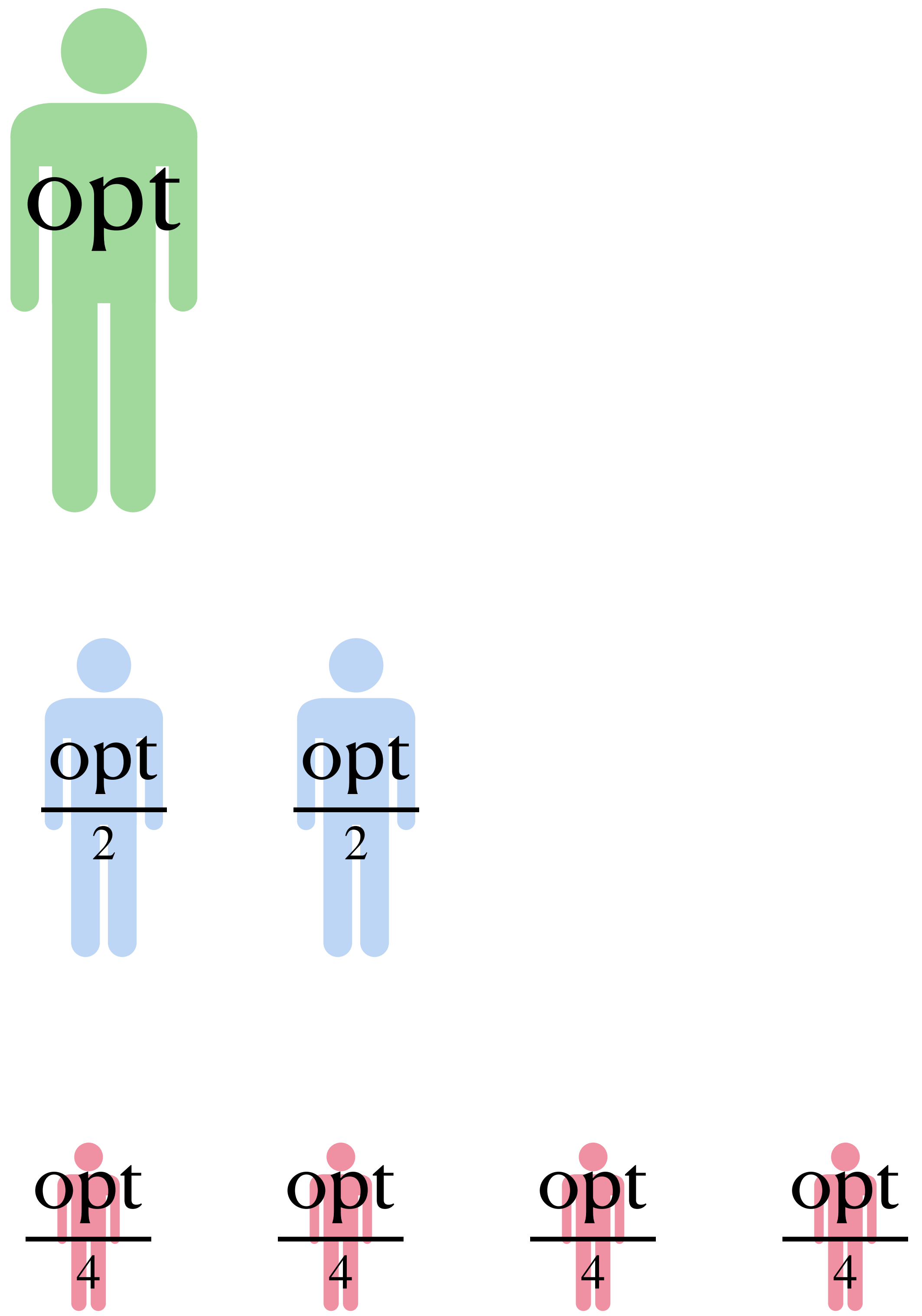
Idea: high cost secretaries arrive first

Batch i:
2^i candidates,
each w/ cost $\frac{\text{opt}}{2^i}$



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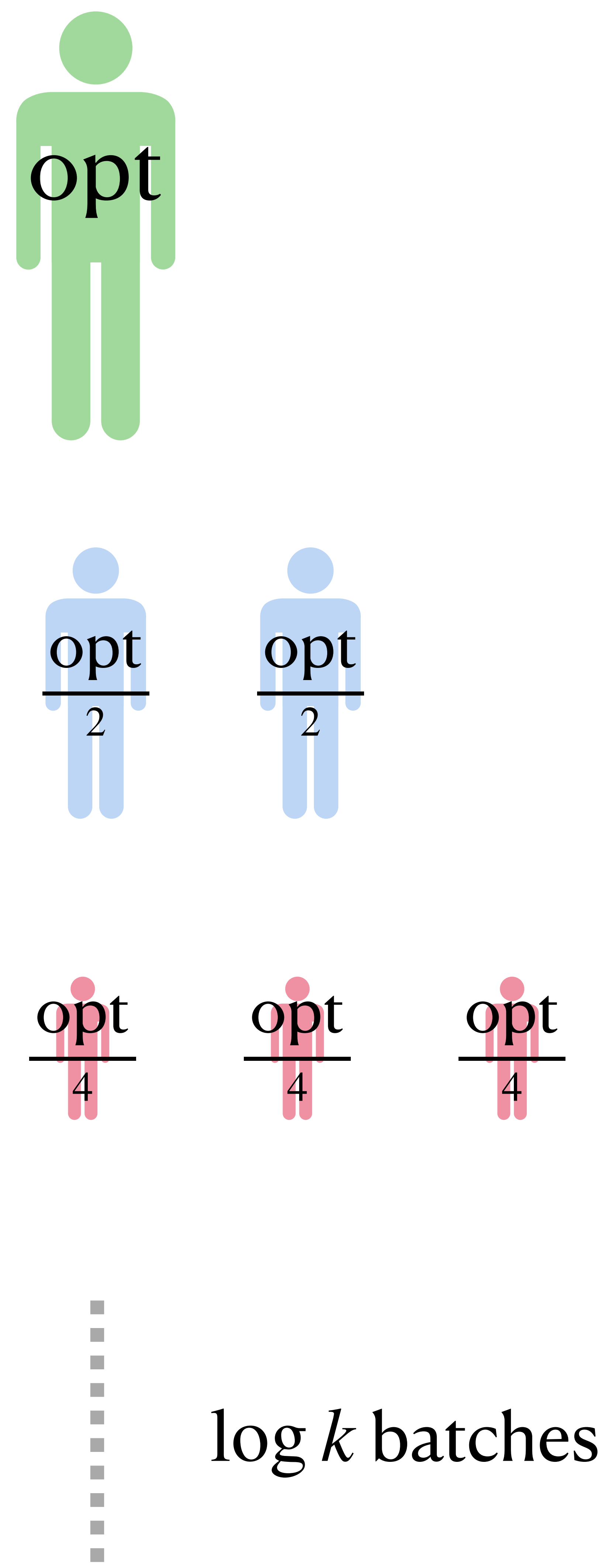

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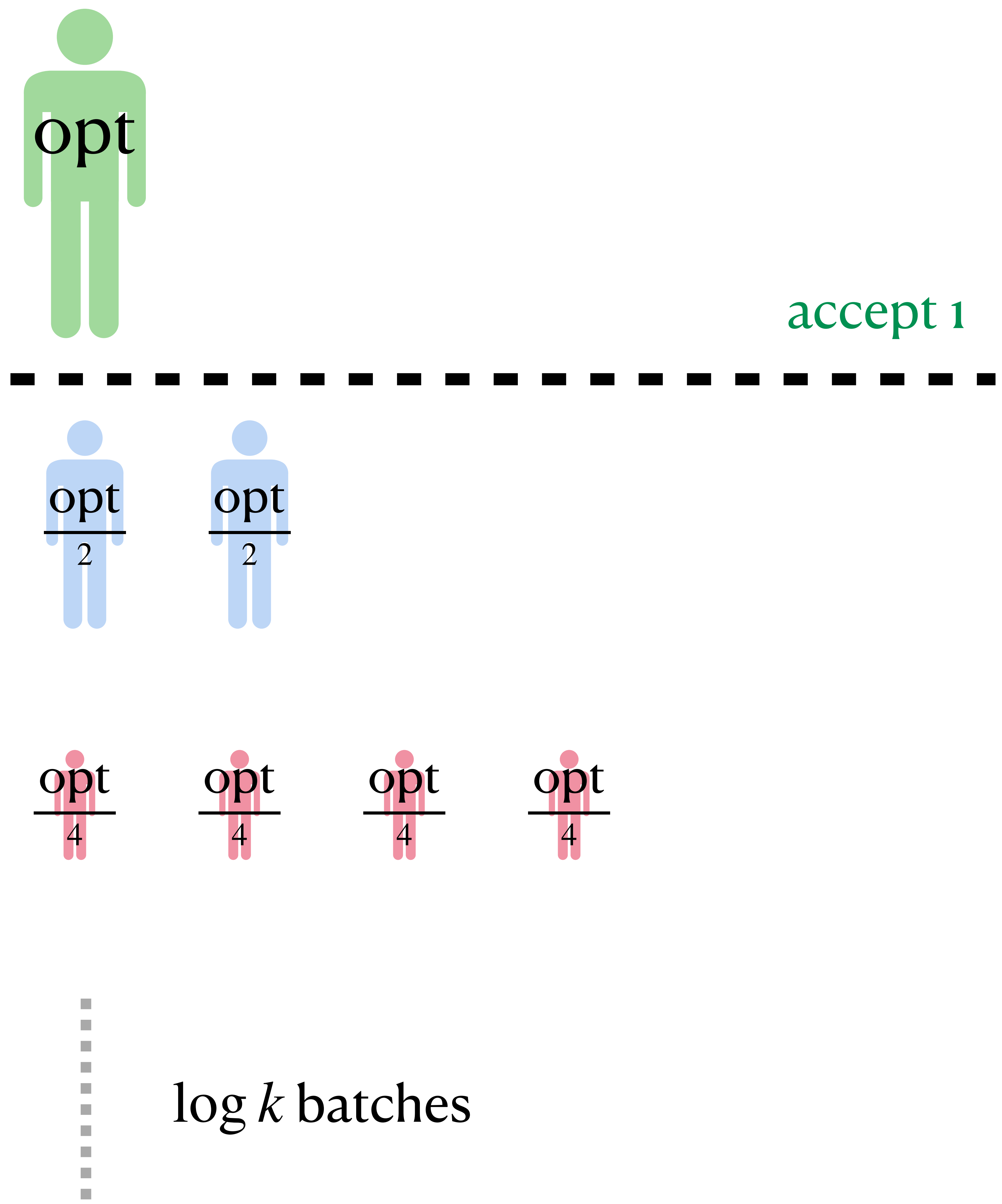

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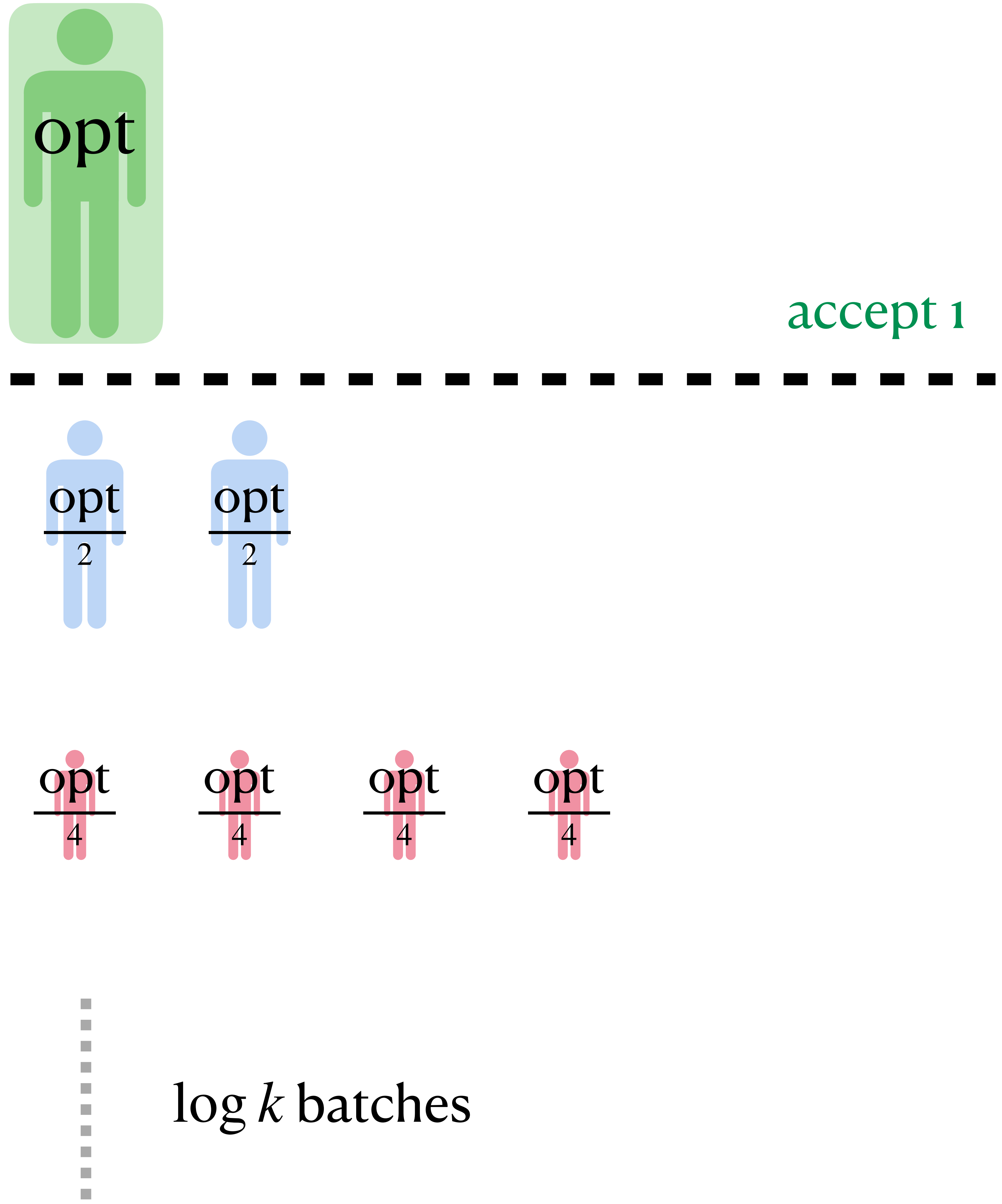

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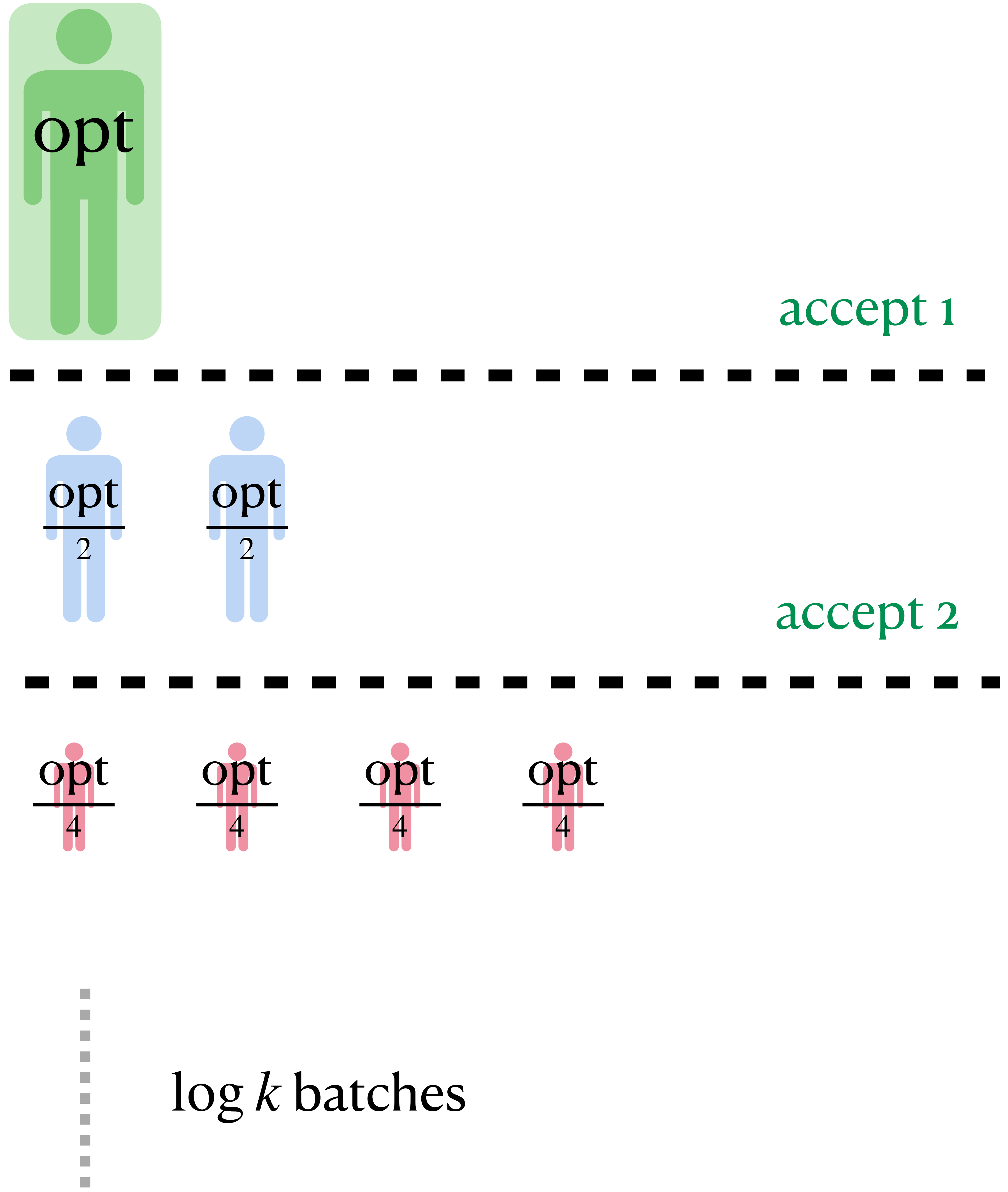

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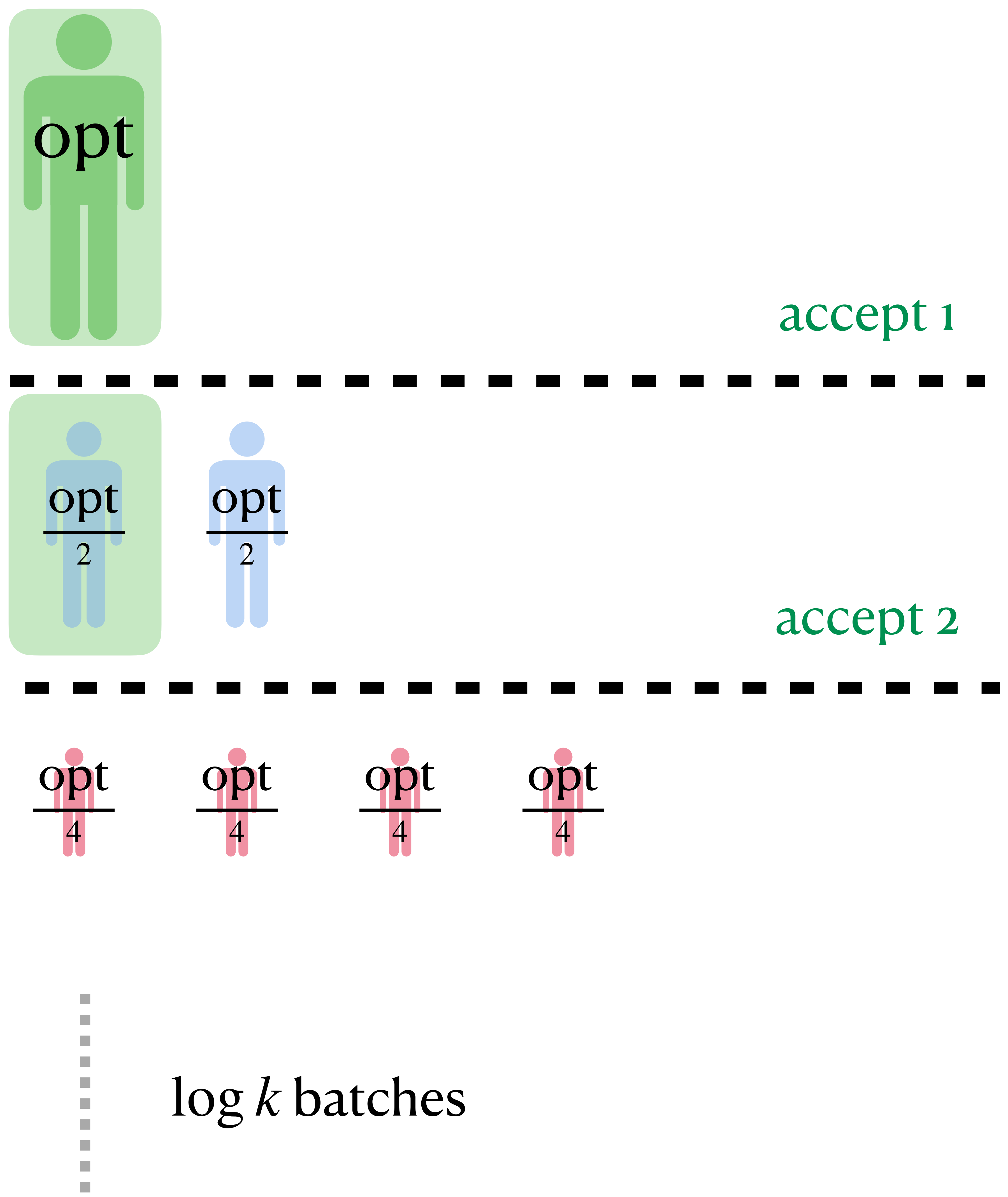

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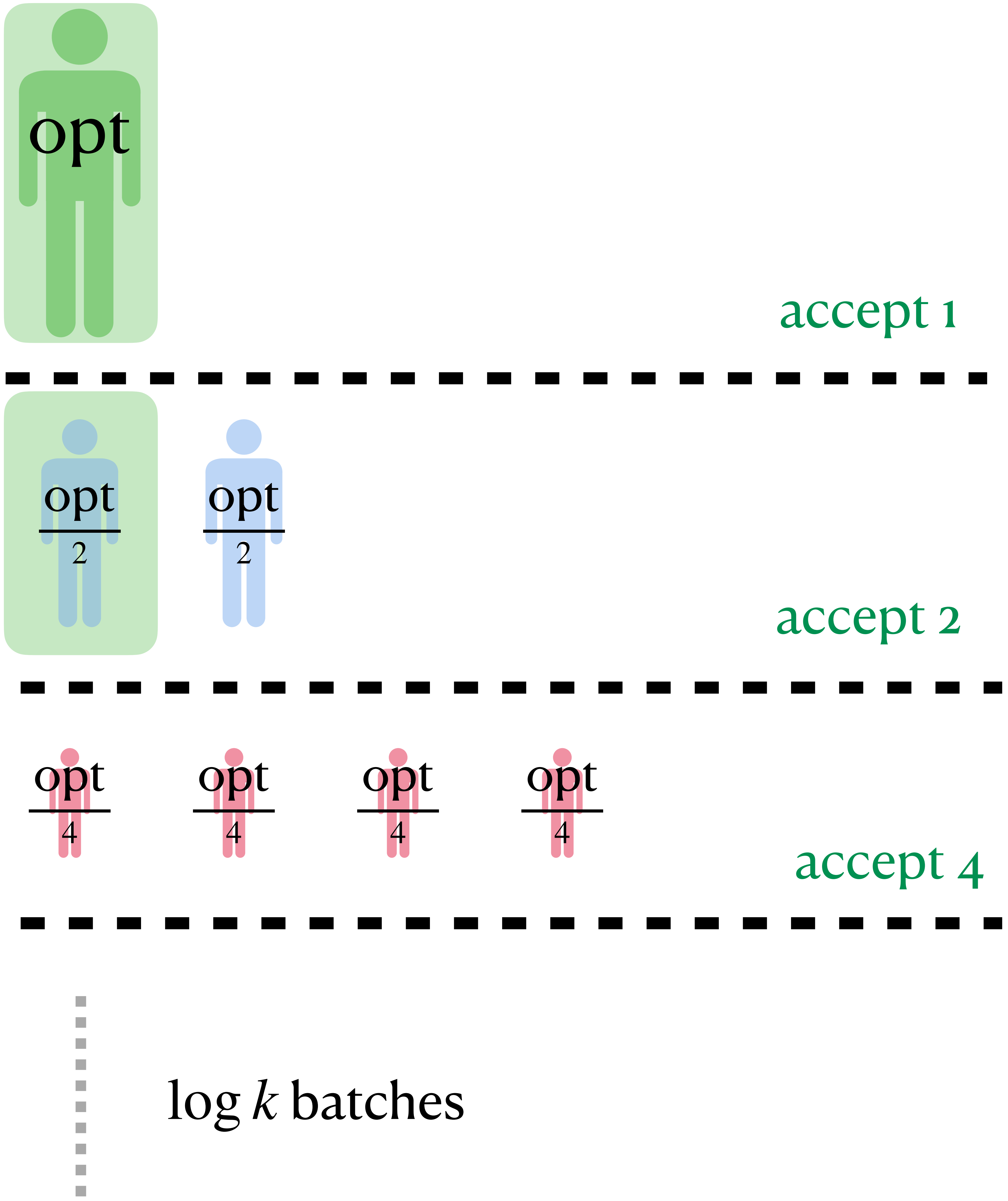

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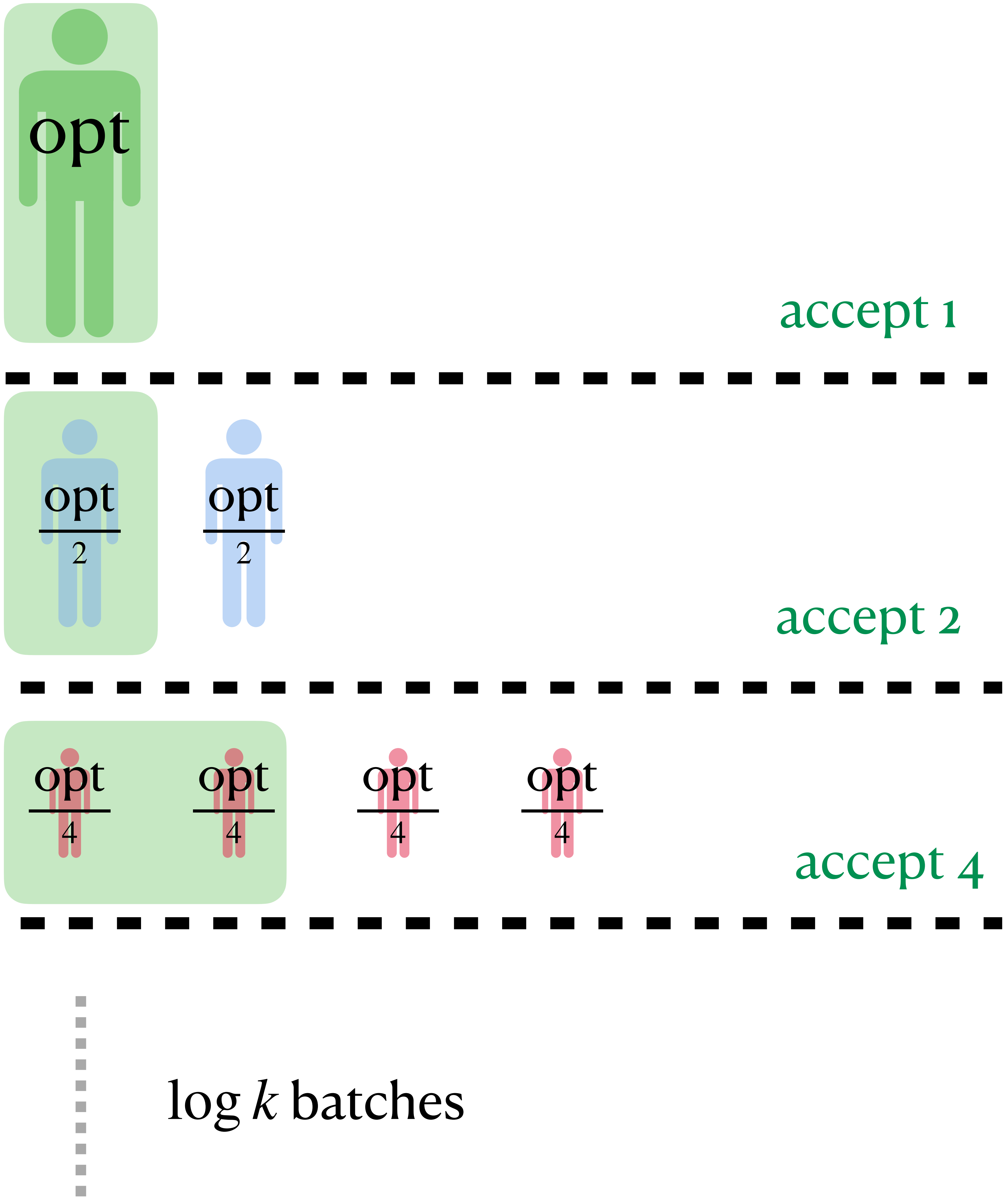

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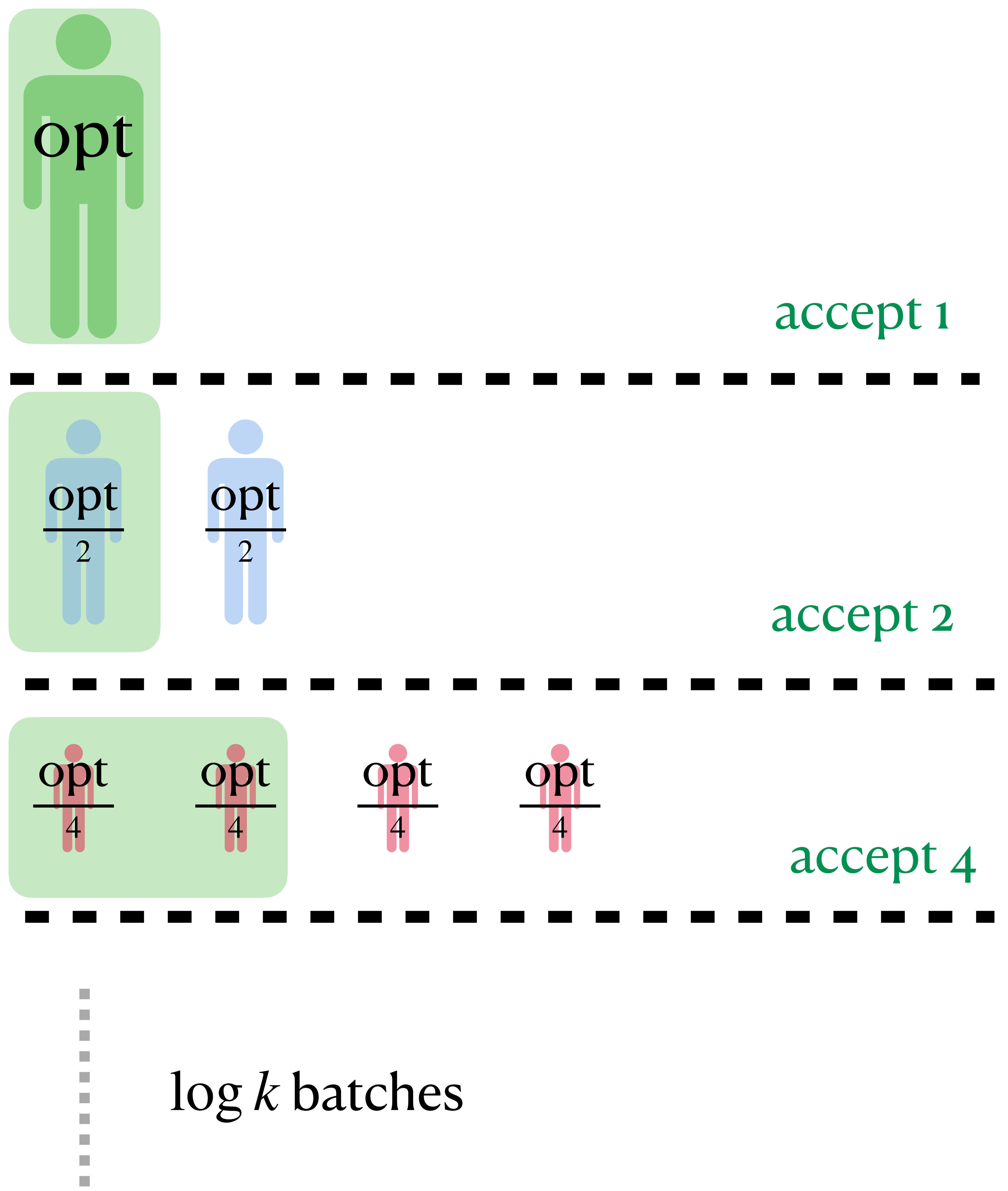
So we accrue $\Omega(B \cdot \log k)$ total cost!



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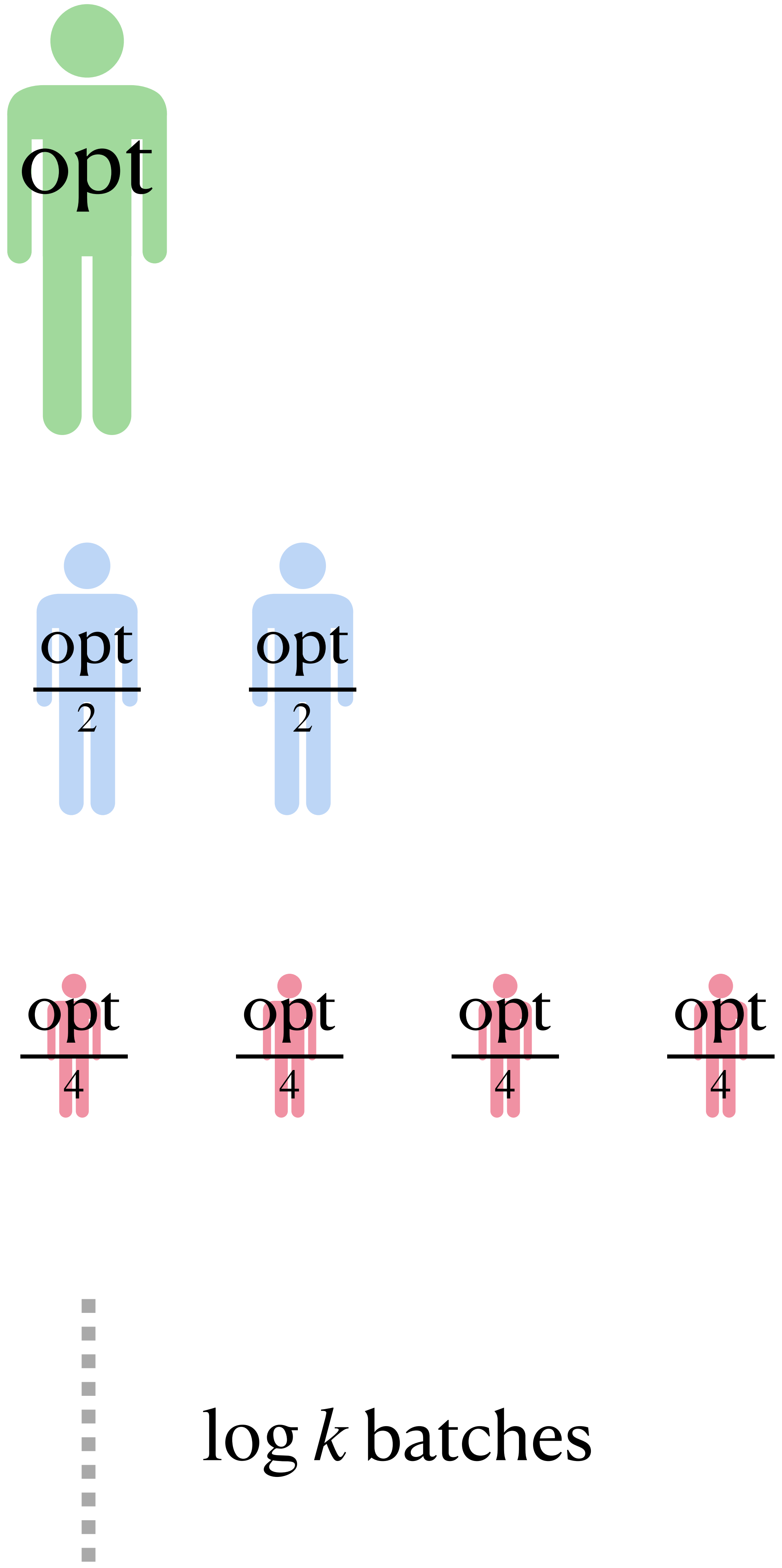
Idea: construct instance such that a random permutation "looks like" adversarial instance with constant probability

Lower Bound: Random Order



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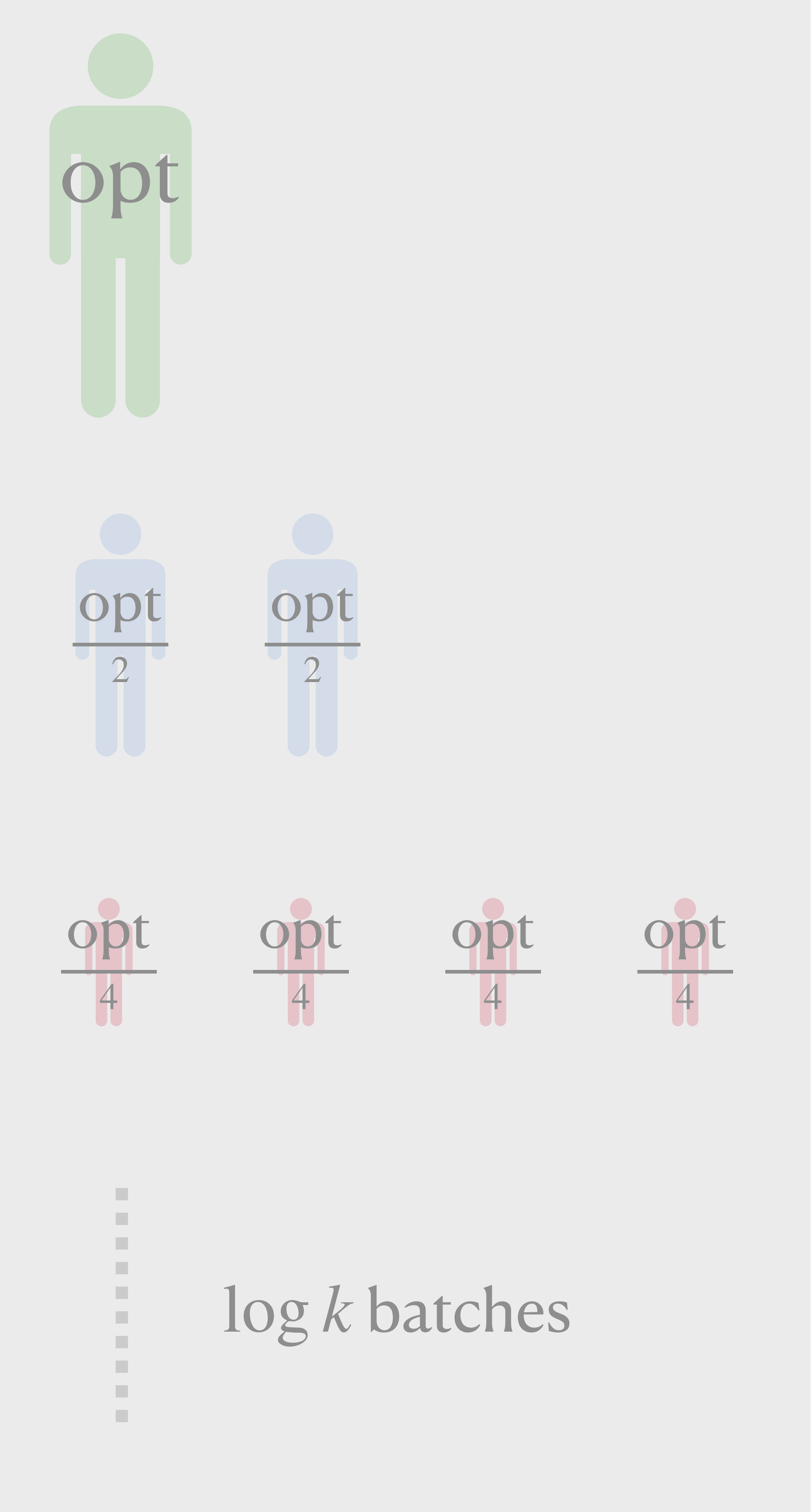

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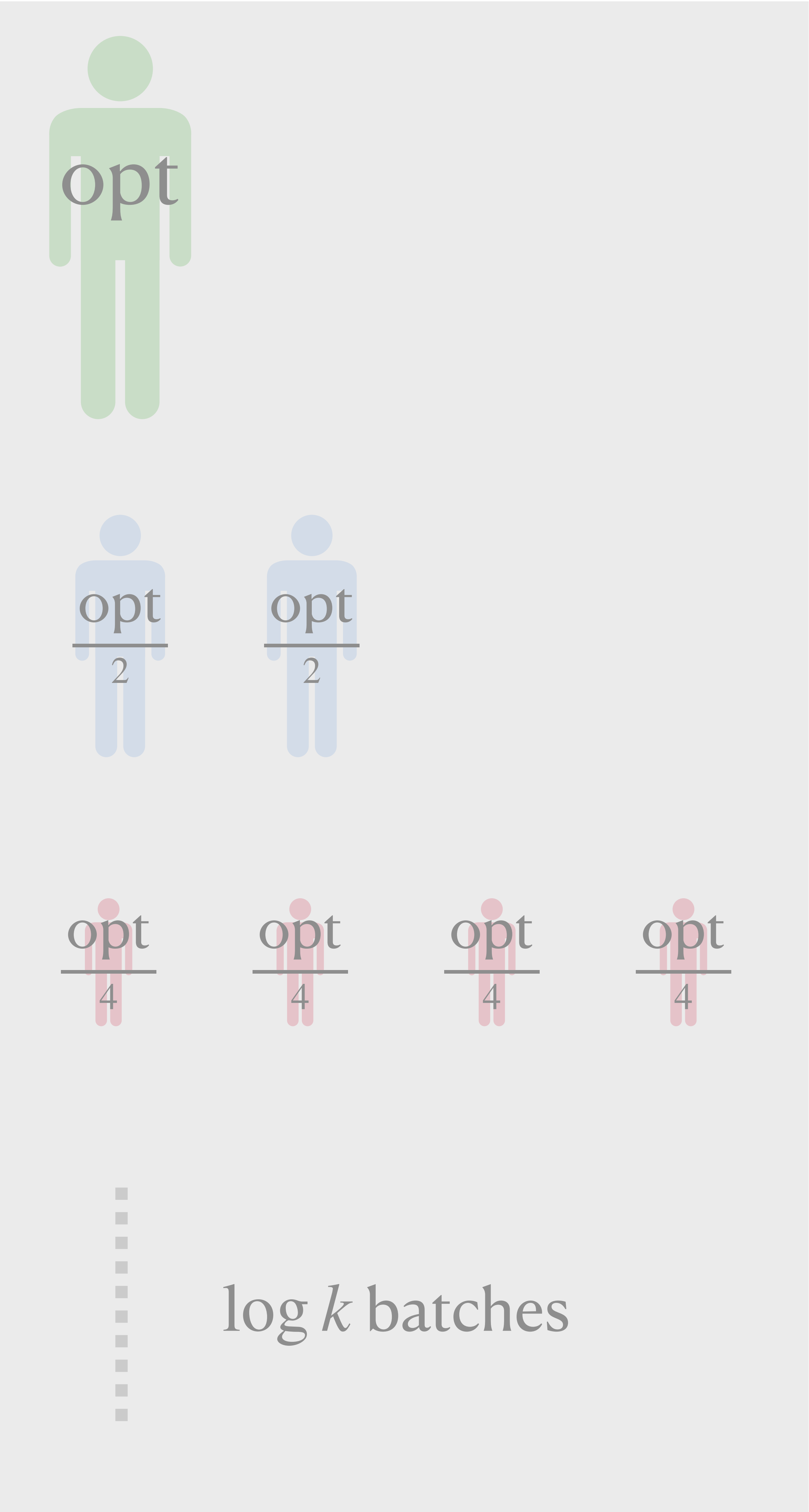
Make many copies of each batch, with earlier batches duplicated more



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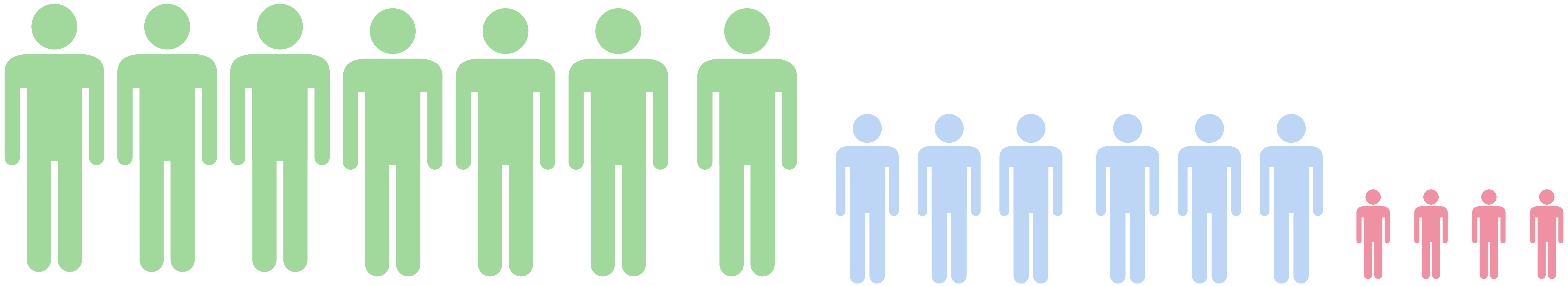

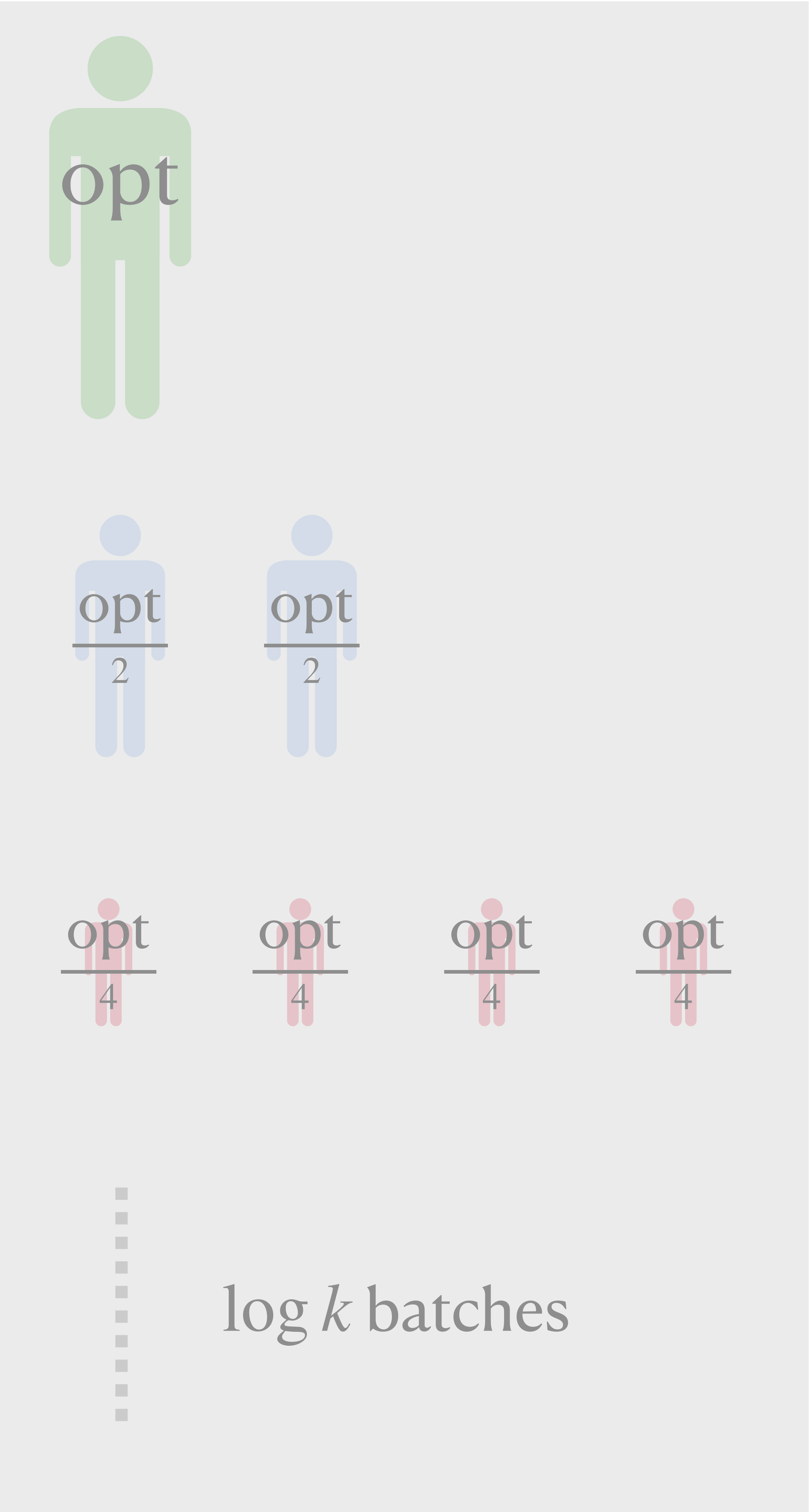

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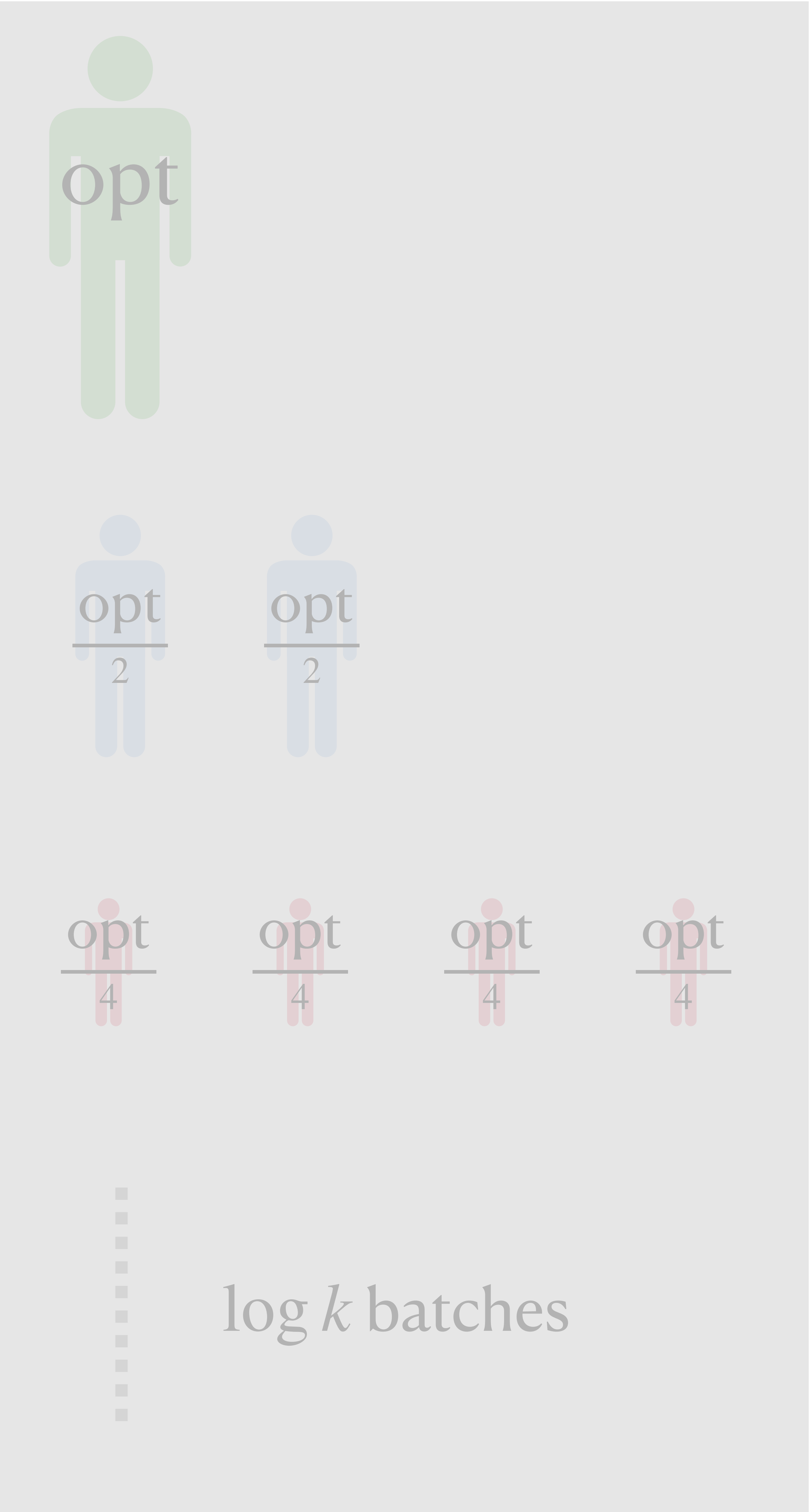

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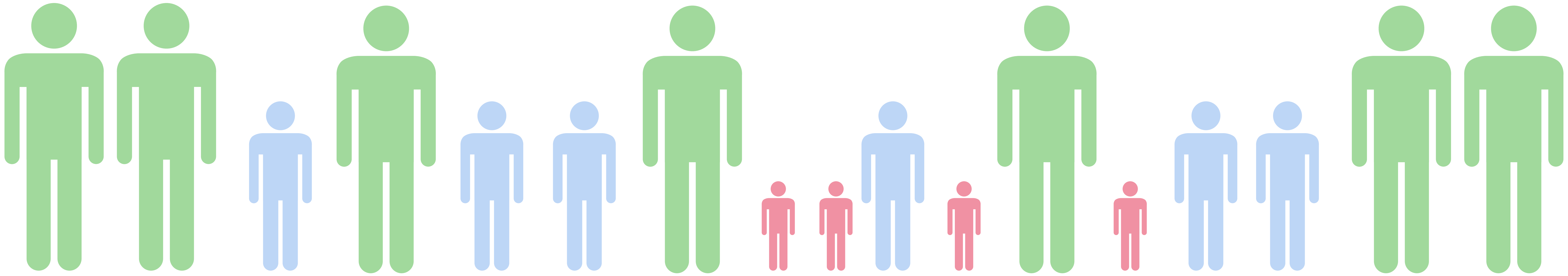

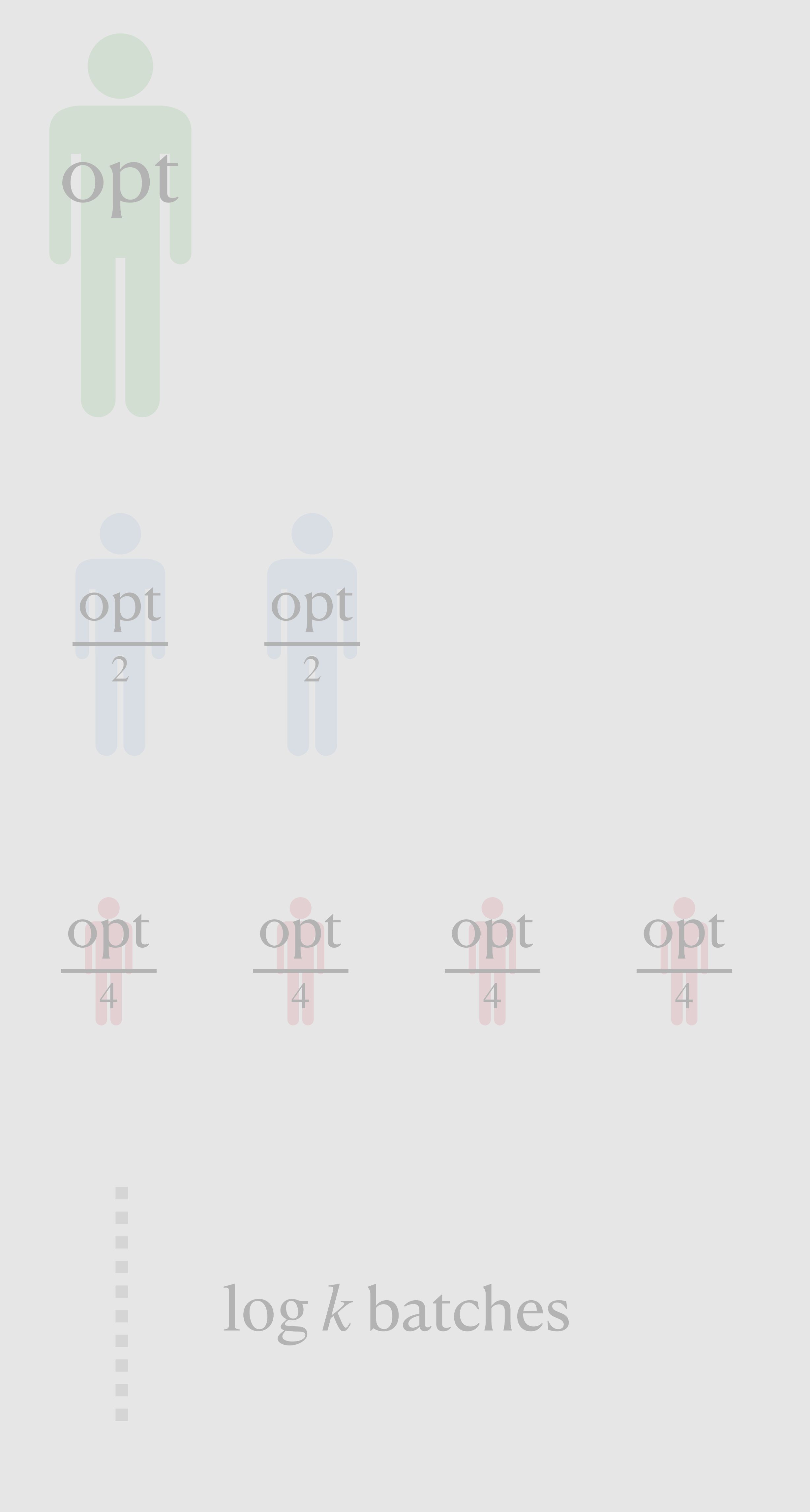

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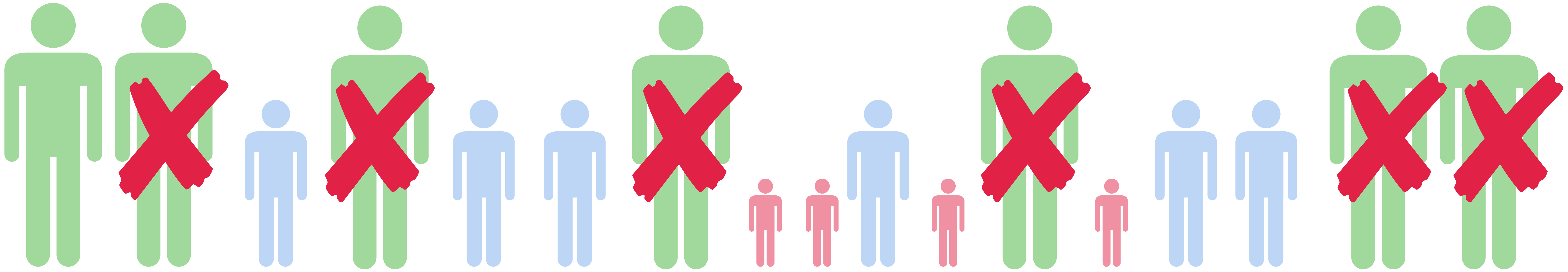

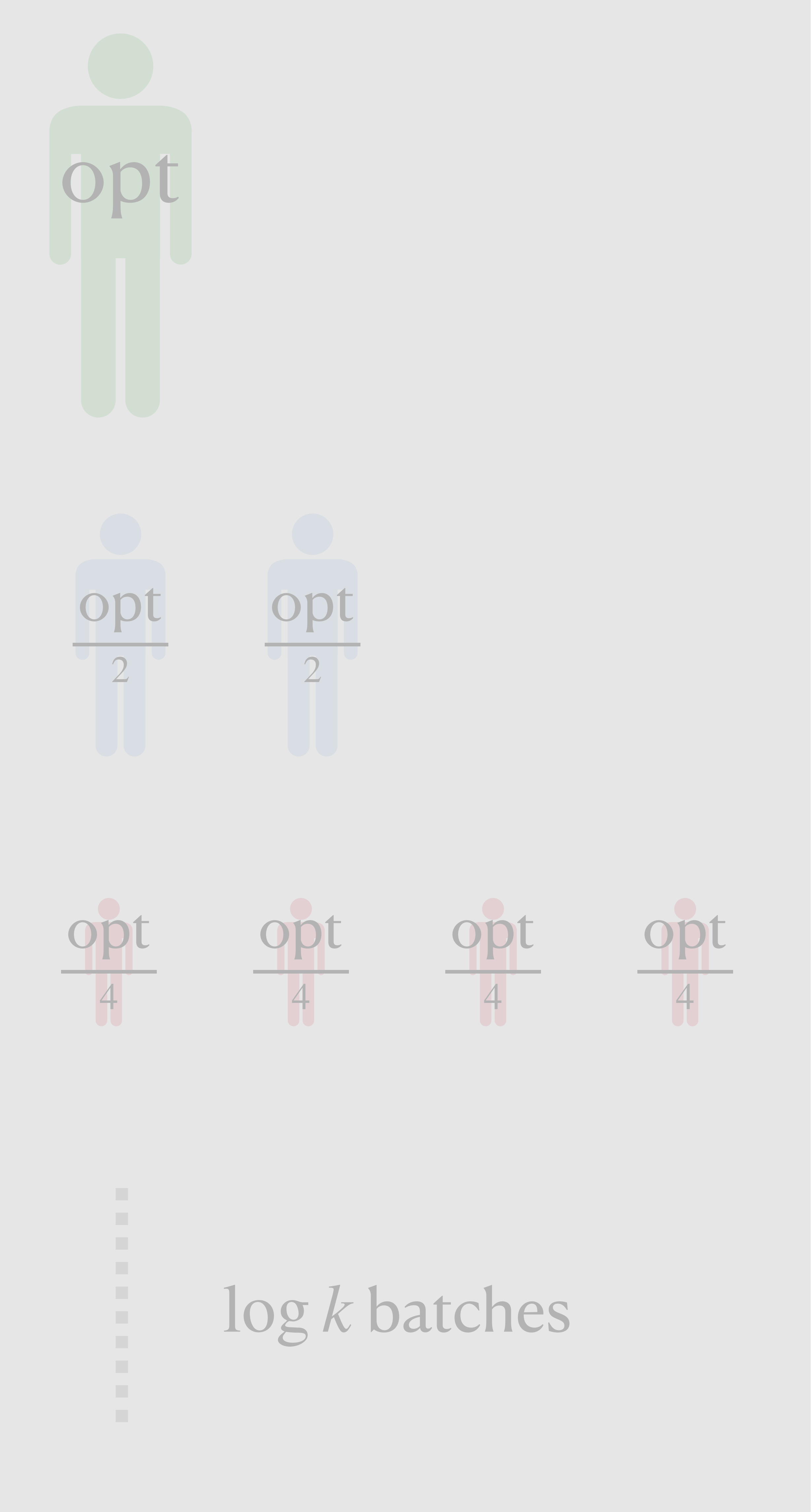

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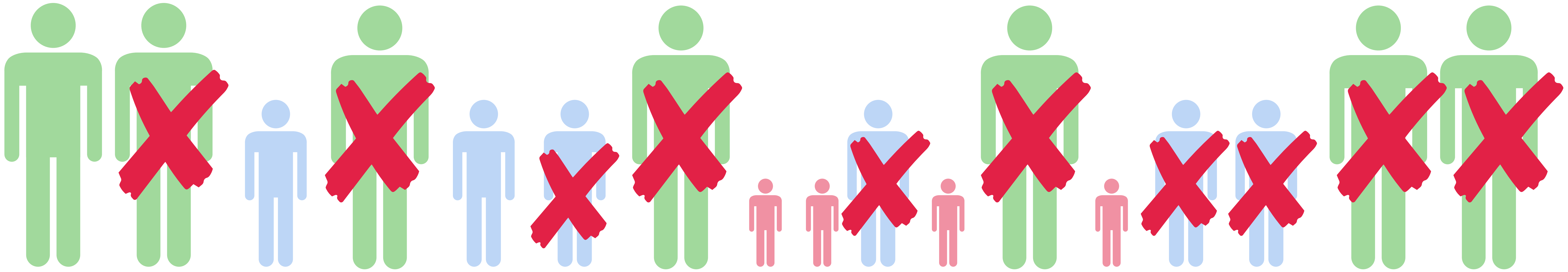

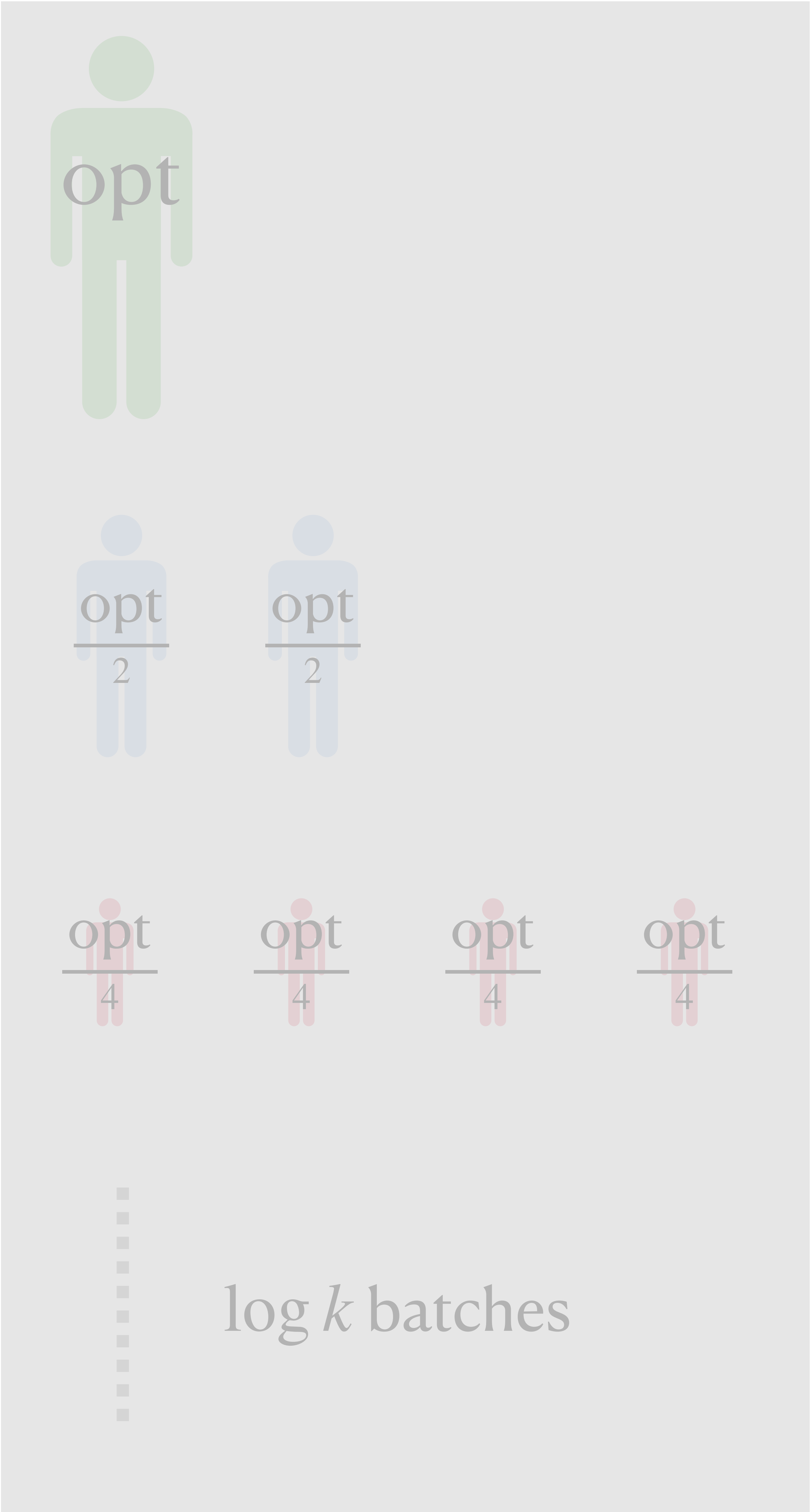

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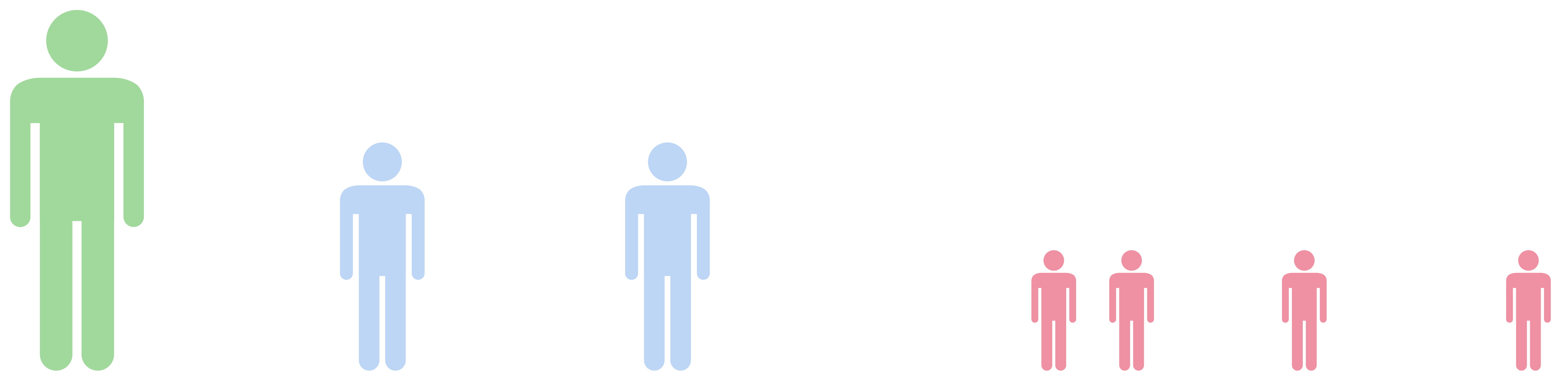

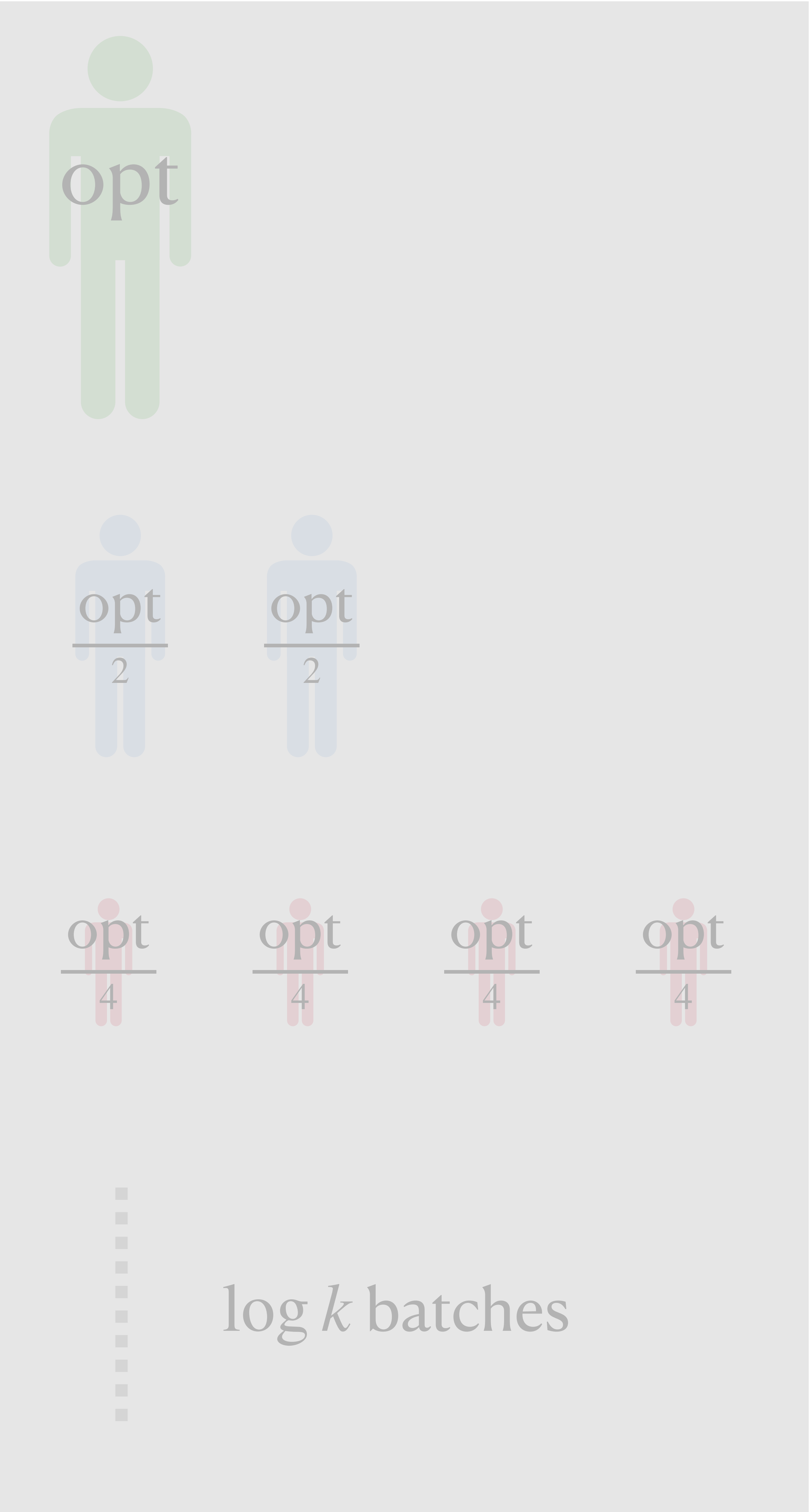

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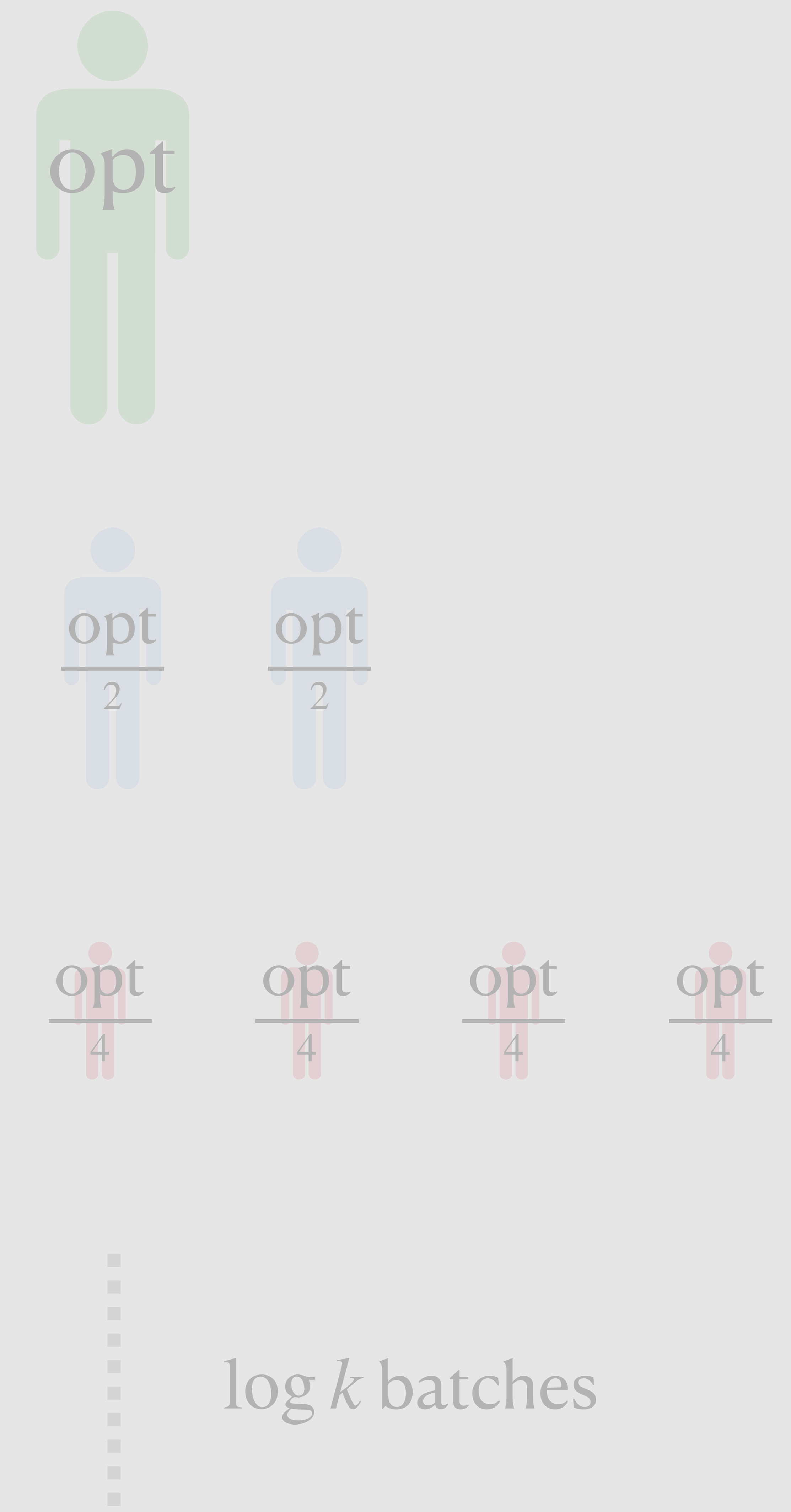

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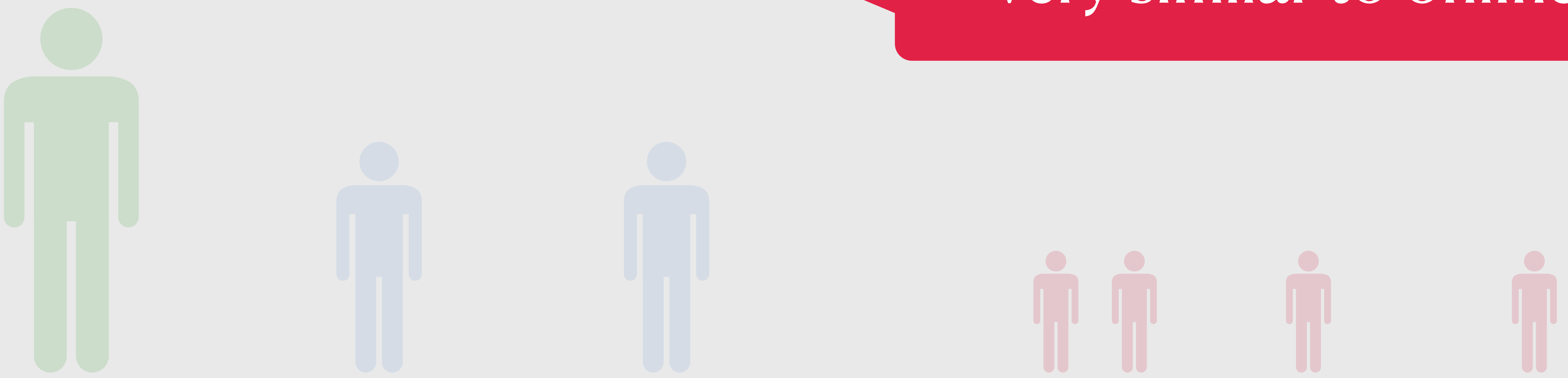
Make many copies of each batch, with earlier batches duplicated more

Very similar to online Steiner tree



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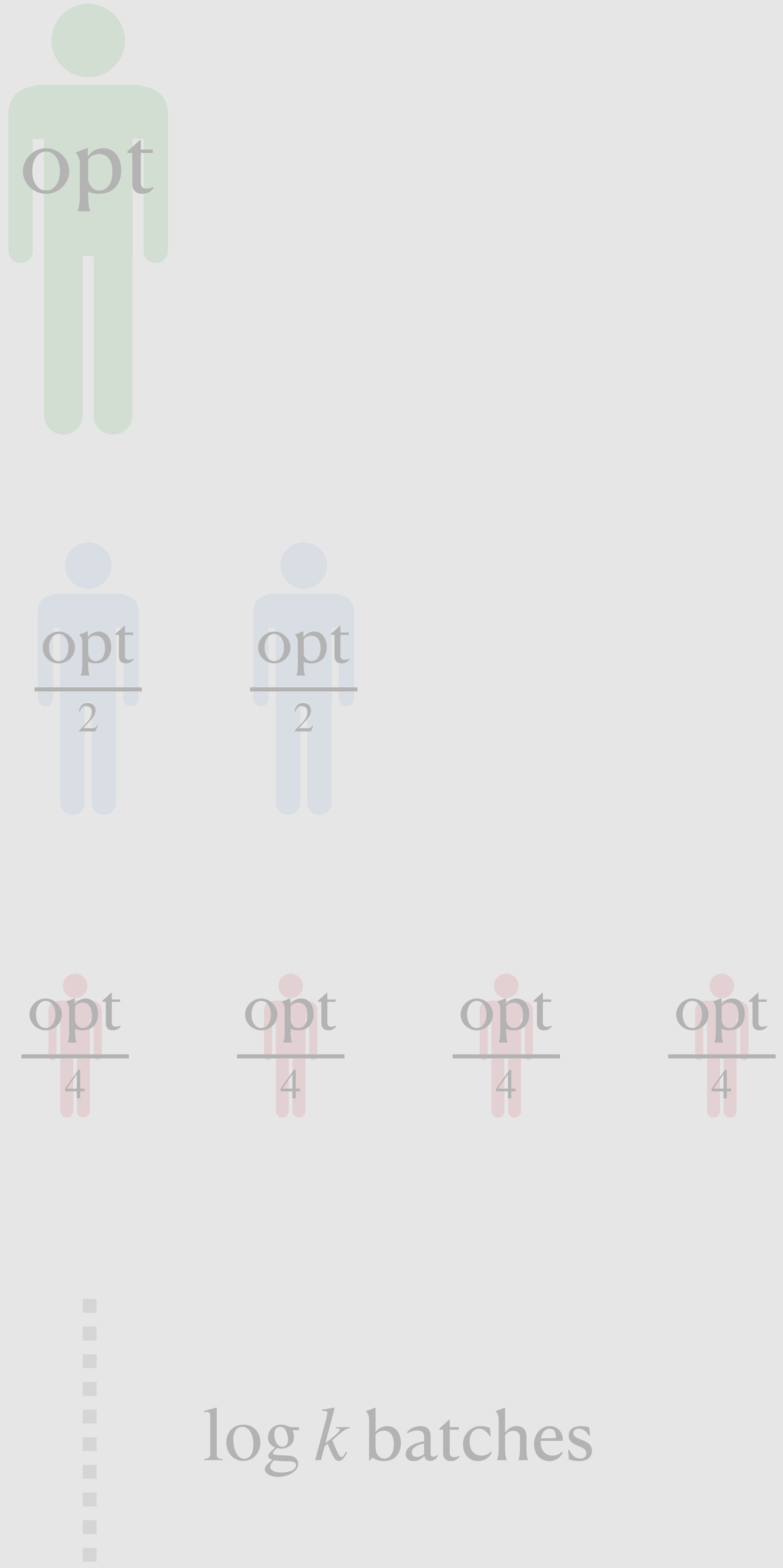
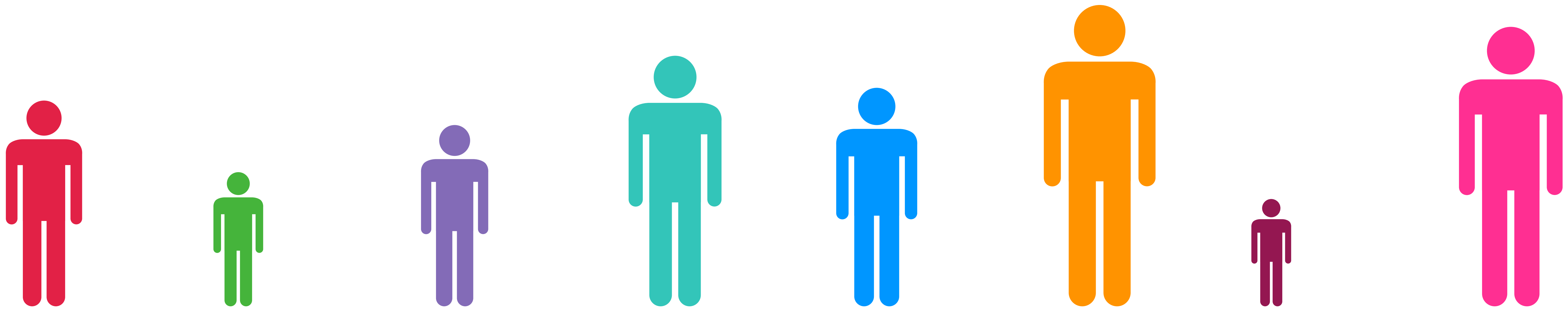


Upper Bound: Adversarial Order

First i candidates: ***(adversarial) arrival order***



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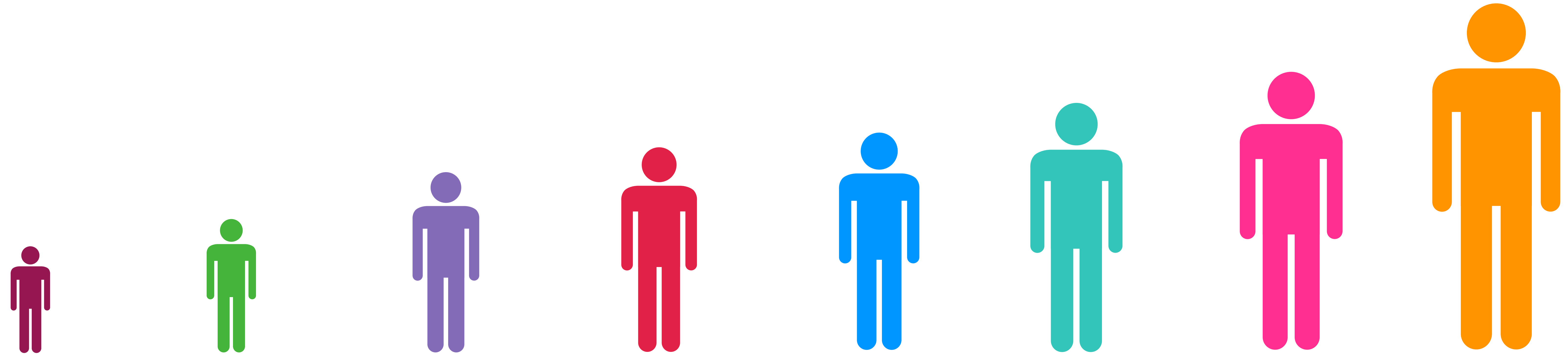

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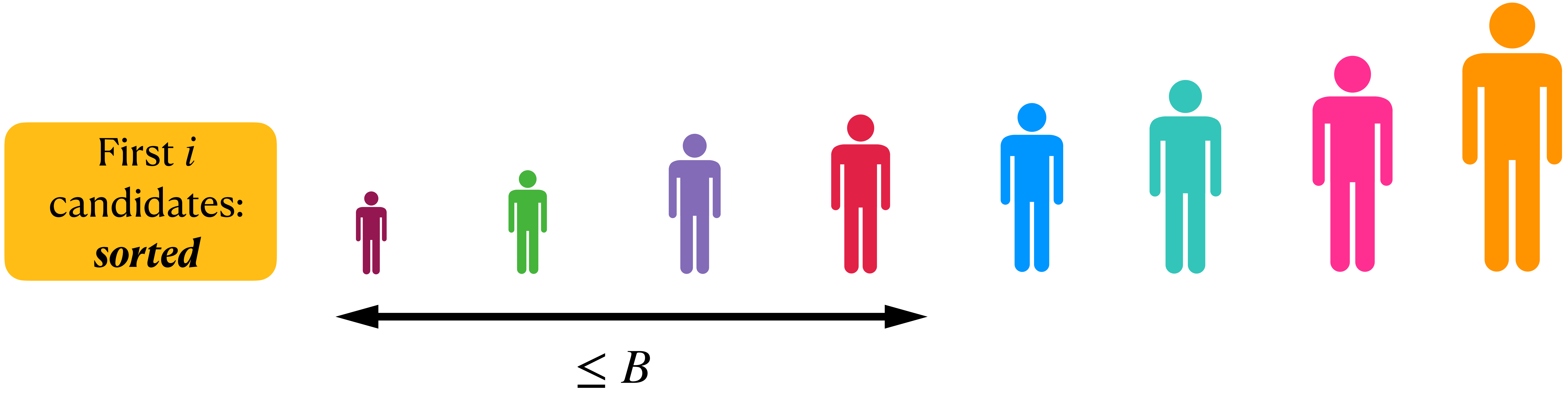

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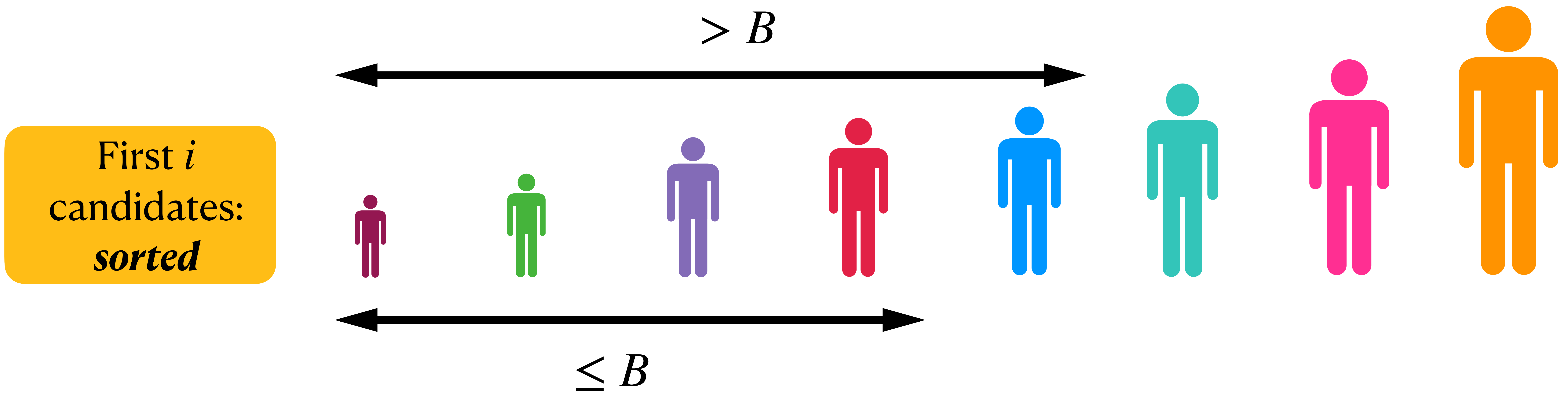

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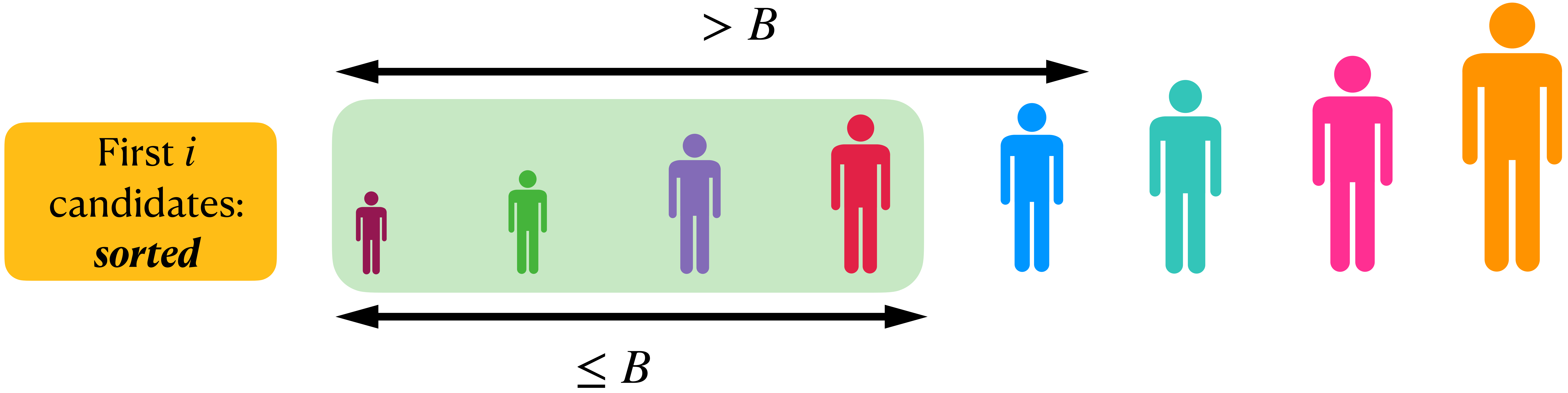

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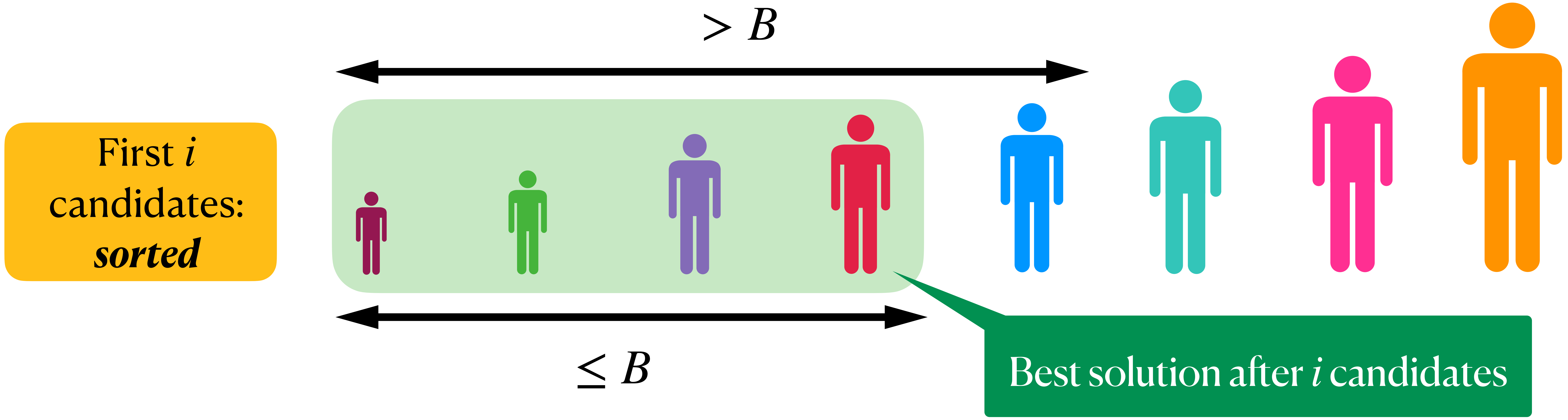

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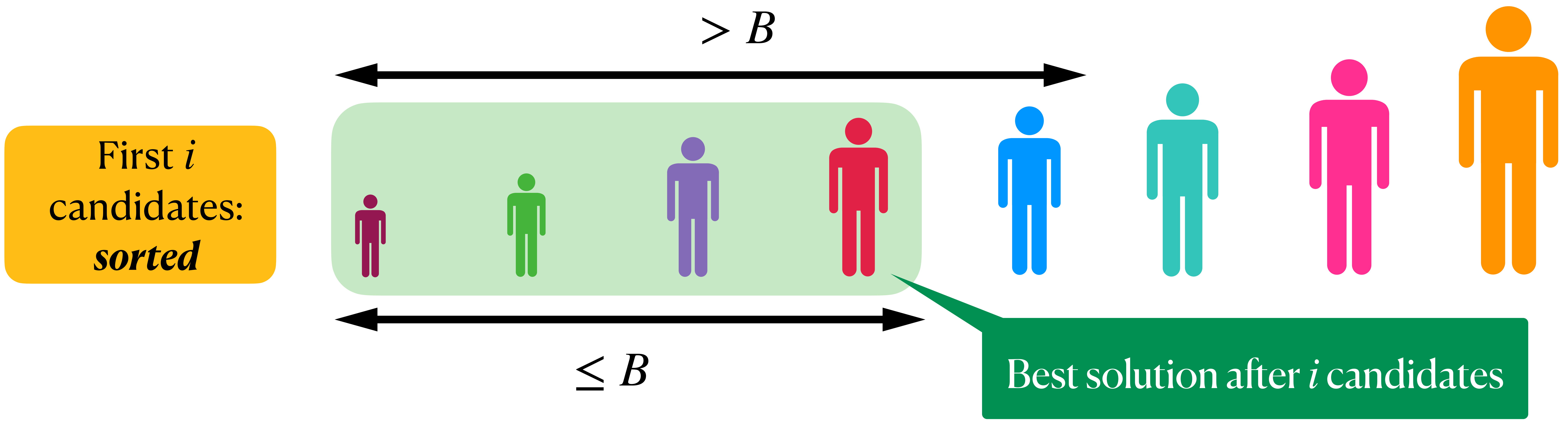
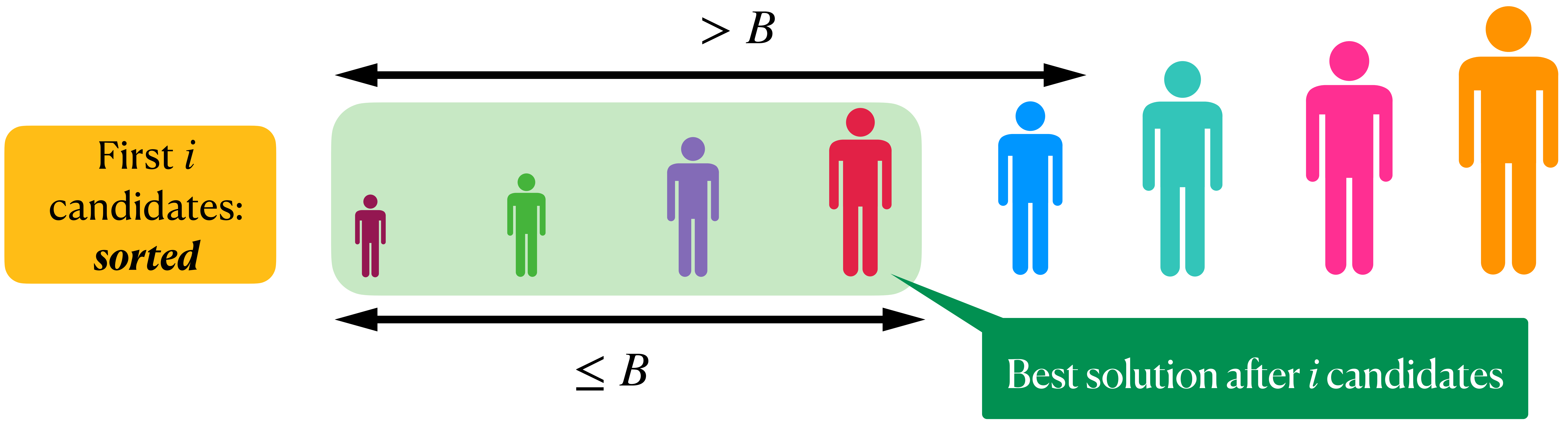


Simplified Cautious Algorithm: Candidate i hired iff candidate i in best solution after i candidates.

Upper Bound: Adversarial Order



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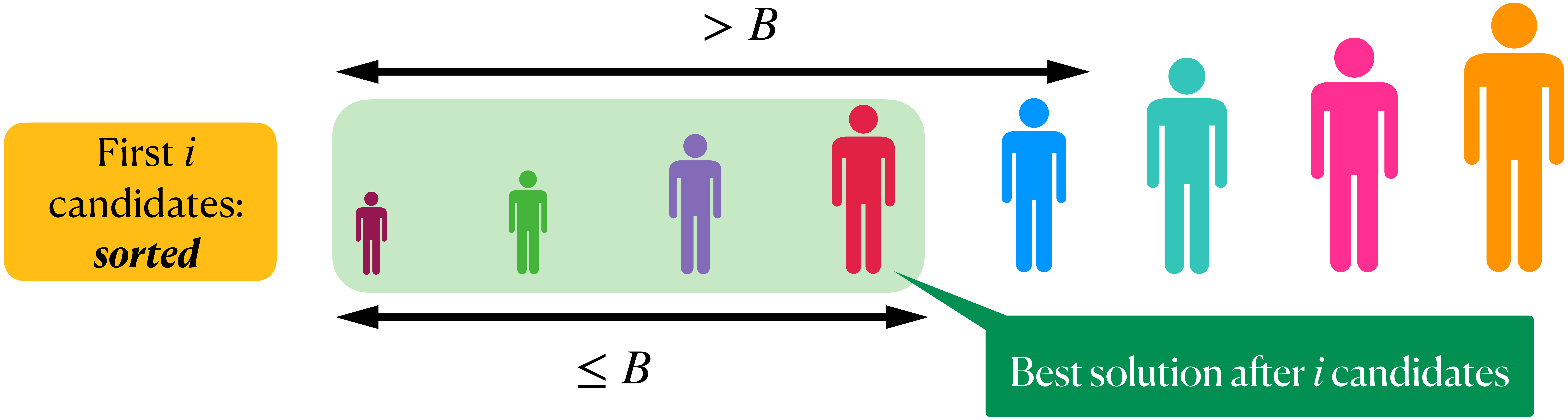

~~**Simplified Cautious Algorithm:** Candidate i hired iff candidate i in best solution after i candidates.~~

Unbounded competitiveness!

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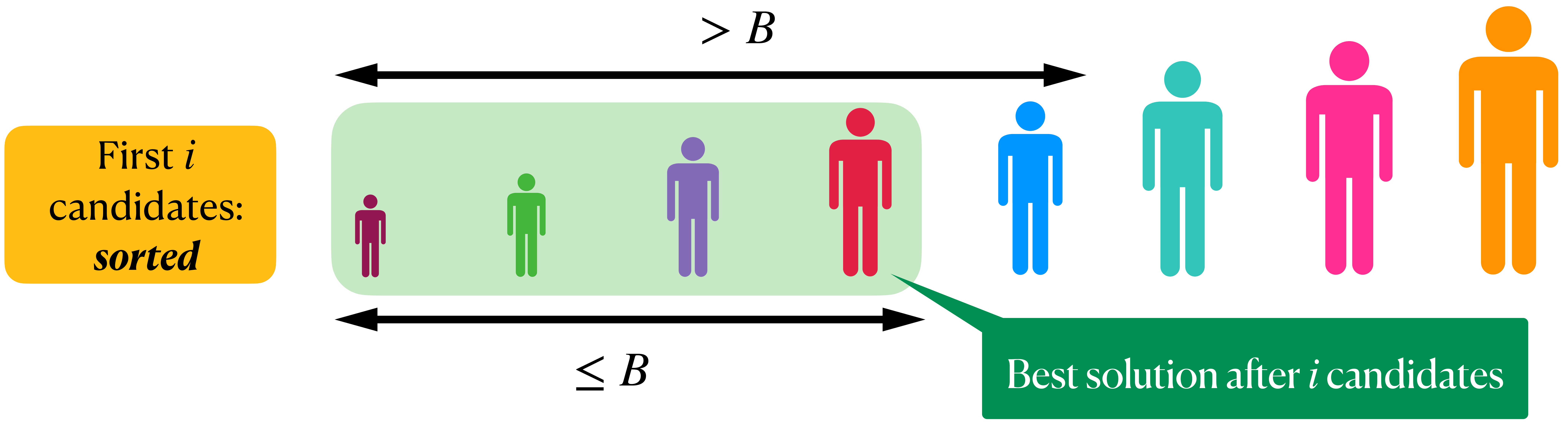

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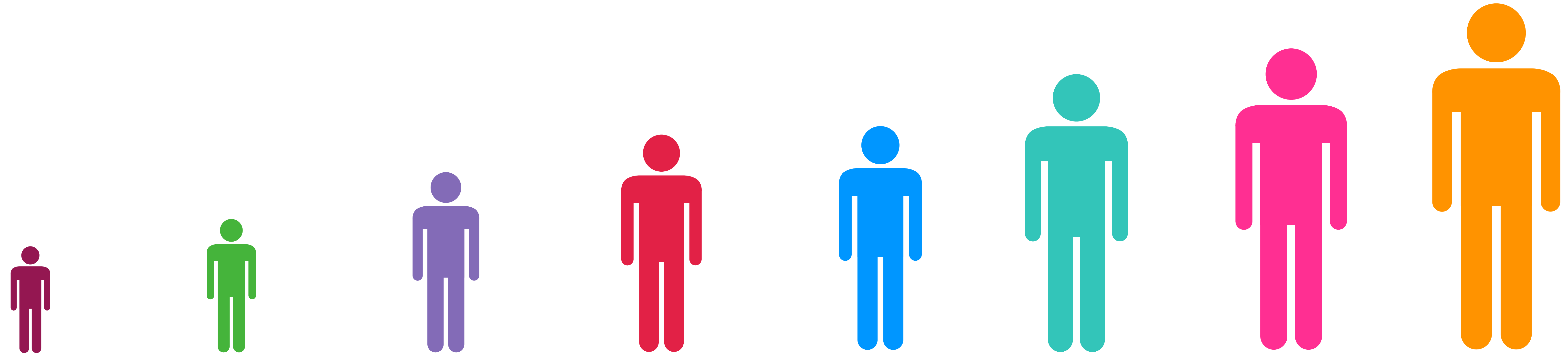
Unbounded competitiveness!

Why?

Treats candidates of similar cost differently.

Upper Bound: Adversarial Order

First i candidates: ***sorted***



~~**Simplified Cautious Algorithm:** Candidate i hired iff candidate i in best solution after i candidates.~~

Unbounded competitiveness!

Why?

Treats candidates of similar cost differently.

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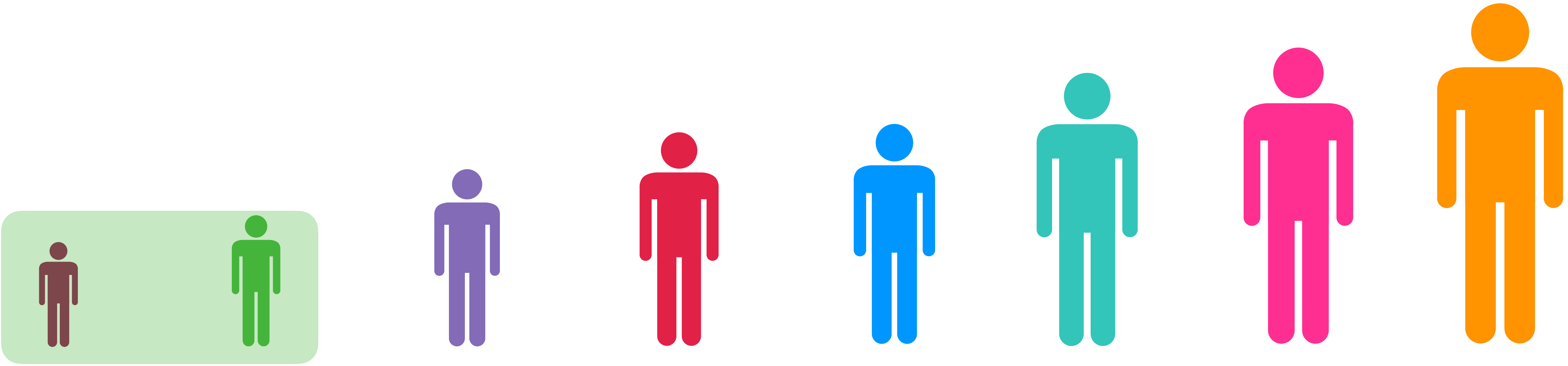

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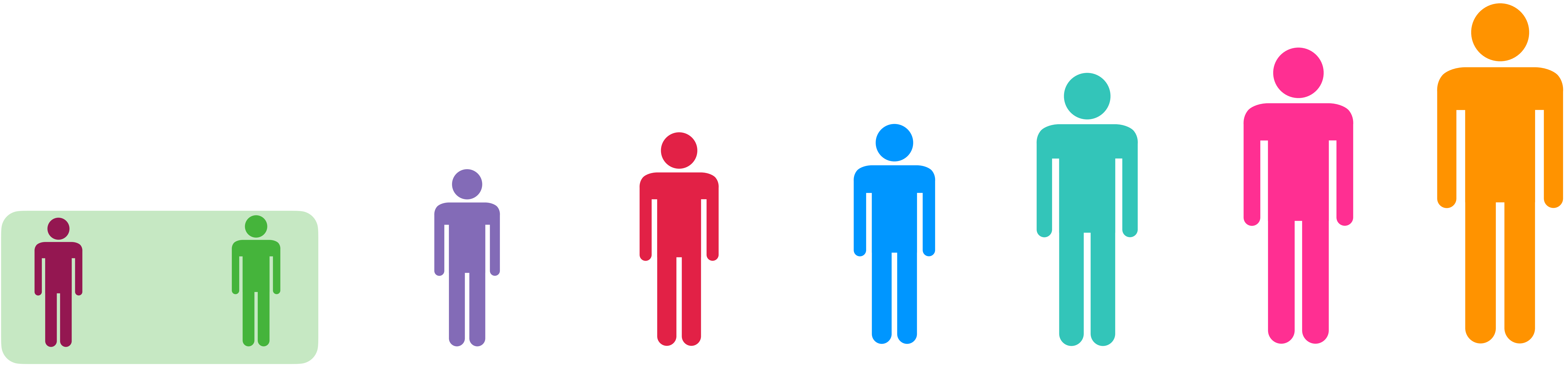

Round up small costs

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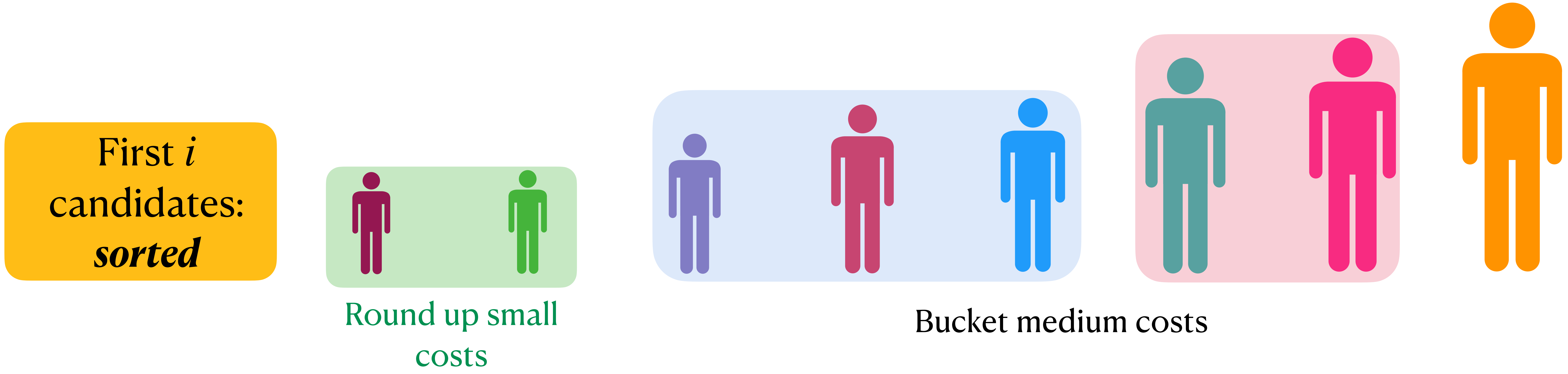

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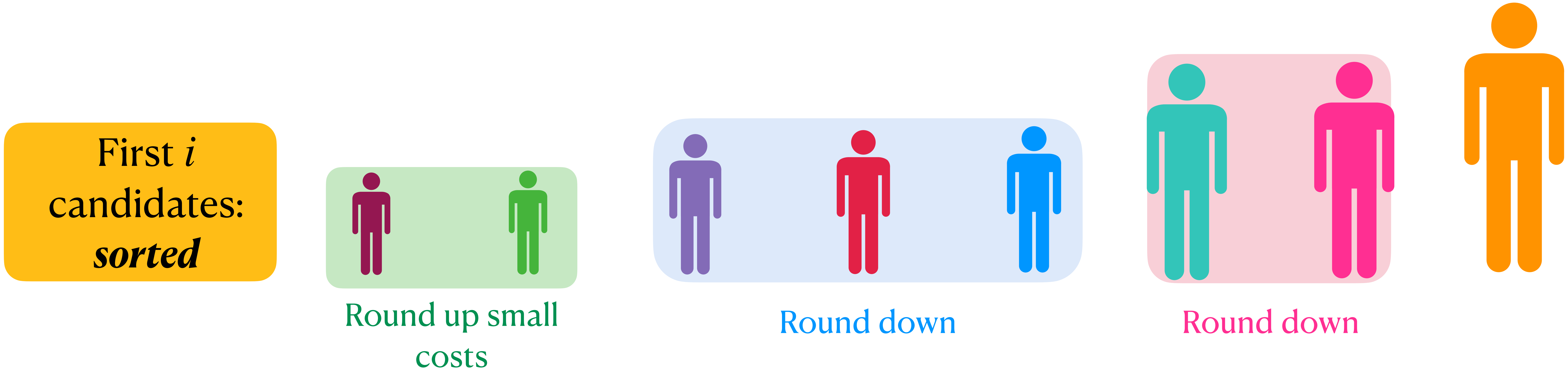

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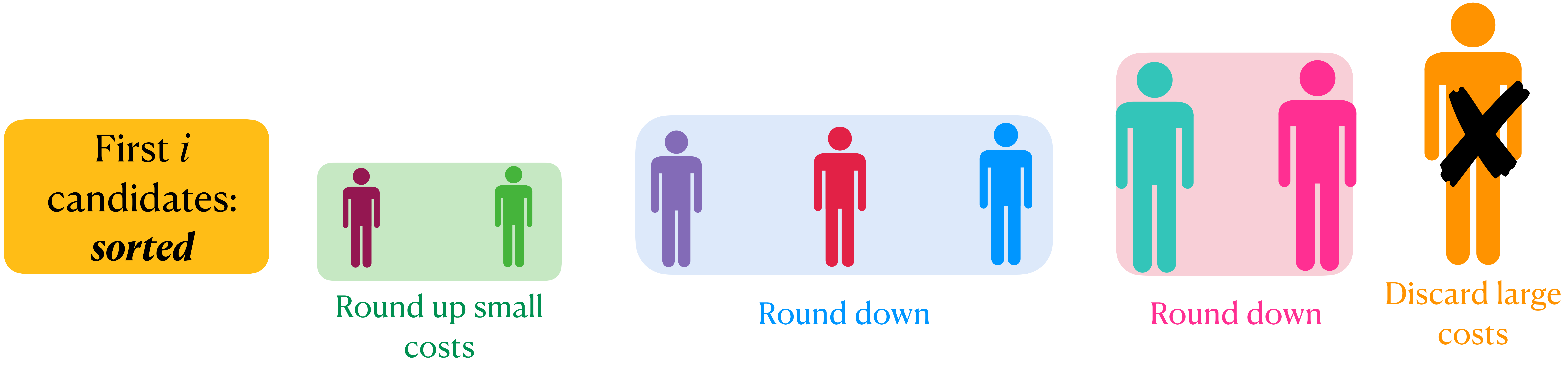

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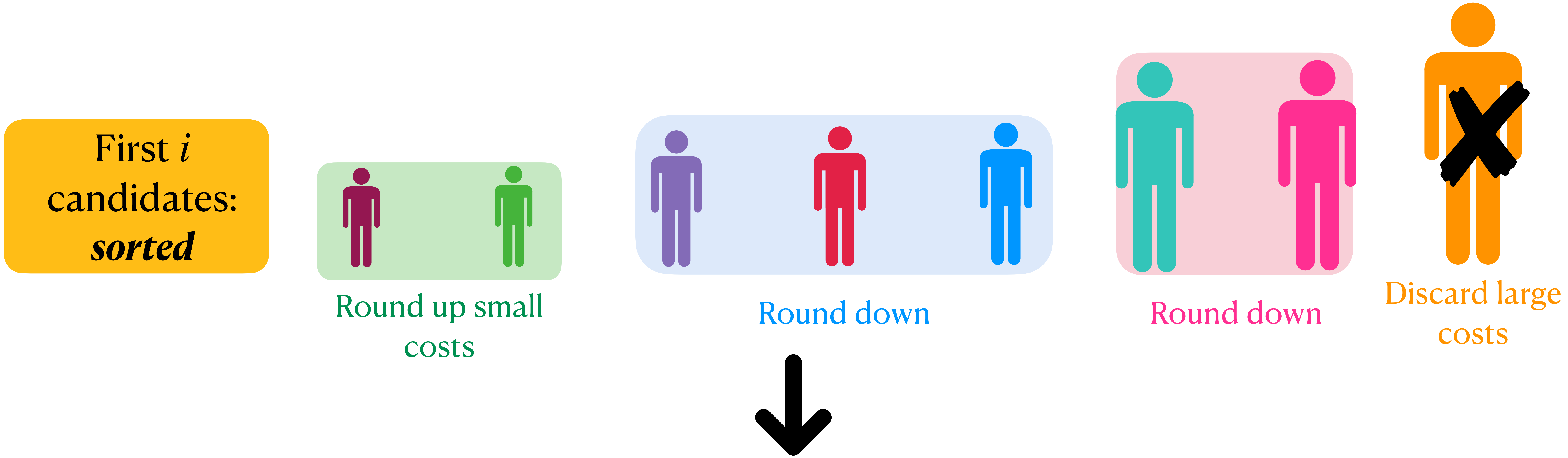

General **Cautious Algorithm:** Candidate i hired iff candidate i in **"best"** solution after i candidates.

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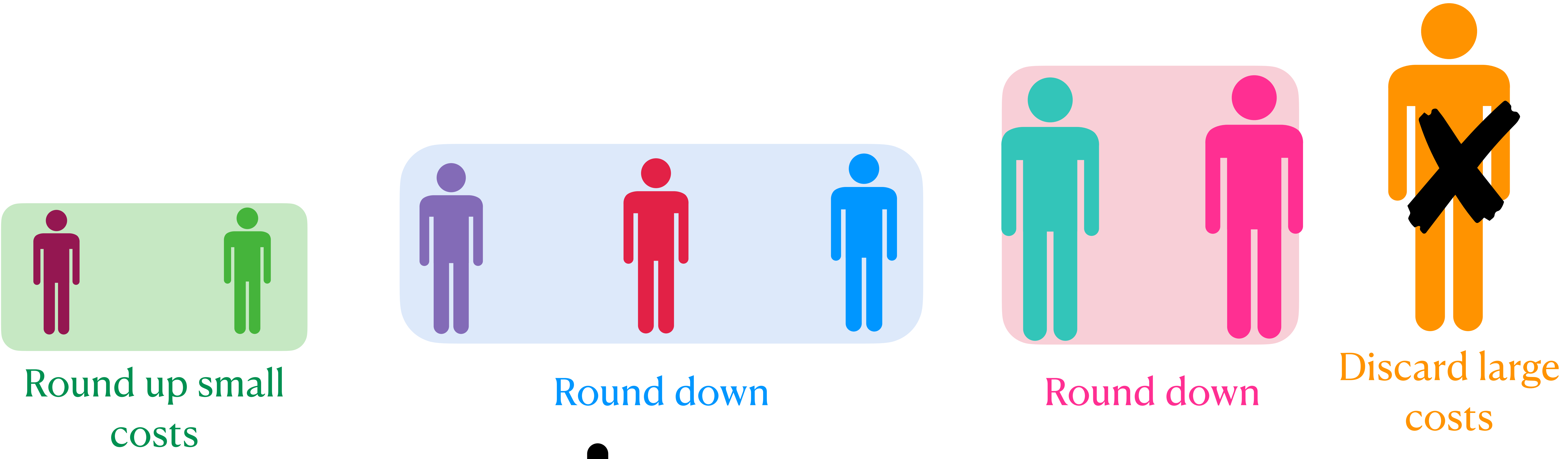

General Cautious Algorithm: Candidate i hired iff candidate i in **“best”** solution after i candidates.

$O(\log k)$**-competitive algorithm**

Future Directions

- Most classic setting: n candidates, maximize expected aggregate value
  - Assume n **is known** AND **random order arrivals**
  - Optimal $1/e$-competitive threshold algorithm for $k = 1$
  - $\left(1 - O\left(1/\sqrt{k}\right)\right)$- competitive algorithm for general k
- Matroid secretary (above: k-uniform matroid)
- Knapsack secretary problems
- Prophet inequality problems
- Secretary with advice
  - Prediction on secretary quality
  - Alternatives to random order: e.g., sample of secretaries

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Minimization variants

Other learning-augmented approaches

Thank You