

Robust Gittins for Stochastic Scheduling

Heather Newman (Carnegie Mellon)
SIGMETRICS 2025

Joint work with: Ben Moseley (Carnegie Mellon), Kirk Pruhs (U. of Pittsburgh), and Rudy Zhou (Microsoft)

Motivation

Motivation

Many stochastic optimization policies assume **perfectly accurate distributions**

Motivation

Many stochastic optimization policies assume perfectly accurate distributions



rich information

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Many stochastic optimization policies assume perfectly accurate distributions



rich information



brittle algorithms

Motivation

Many stochastic optimization policies assume ~~perfectly accurate distributions~~ unrealistic



rich information



brittle algorithms

Motivation

Many stochastic optimization policies assume ~~perfectly accurate distributions~~ ^{unrealistic}

Motivation: develop stochastic optimization algorithms that are **robust to imperfect predicted distributions**

rich information

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Motivation

Many stochastic optimization policies assume ~~perfectly accurate distributions~~ unrealistic

Motivation: develop stochastic optimization algorithms that are **robust to imperfect predicted distributions**

rich information

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Today:

preemptively schedule stochastic jobs on single machine
to minimize total completion time (no release dates)

Related Work

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M/G/1 queue

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M/G/1 queue

- [Scully, Grosof, & Mitzenmacher '22](#): give scheduler **stochastic estimate** z_j of true size s_j where $(s_j, z_j) \sim (S, Z)$ and $z_j \in [\beta \cdot s_j, \alpha \cdot s_j] \rightarrow$ compare against SRPT

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Scheduling with predictions (non-stochastic setting)

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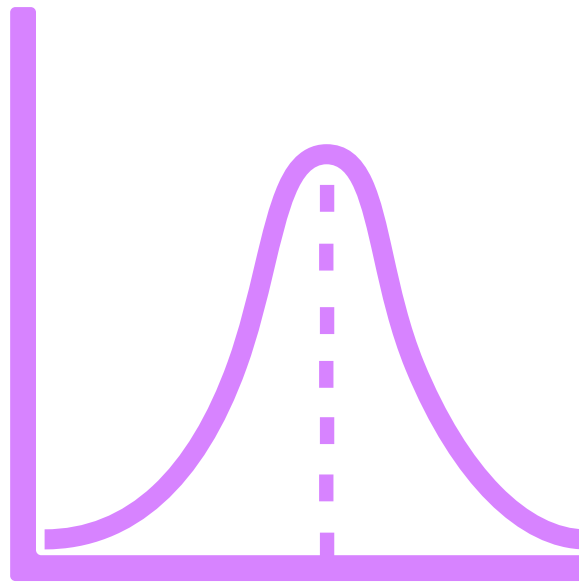
Non-scheduling problems

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- [Kim & Lim '16](#): Multi-armed bandits

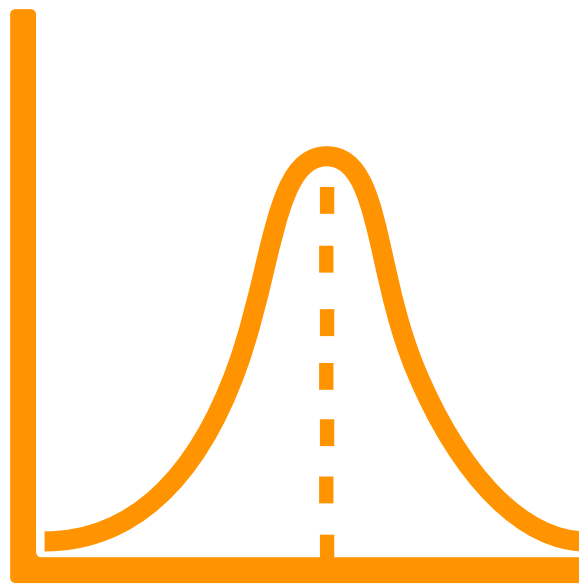
Problem Definition

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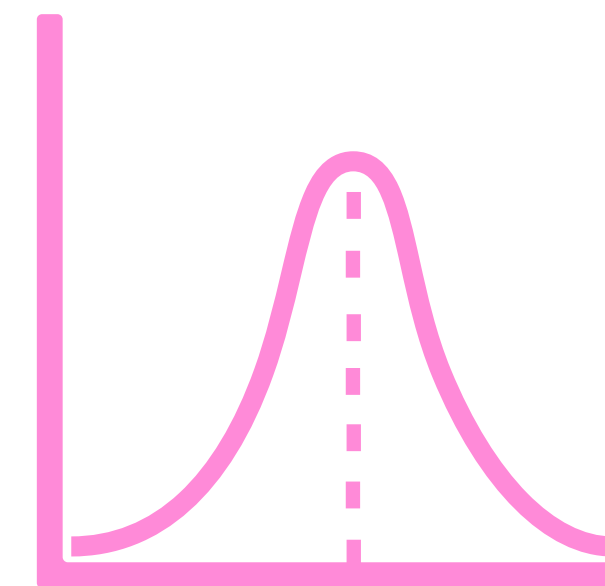
\mathcal{D}_1



\mathcal{D}_2



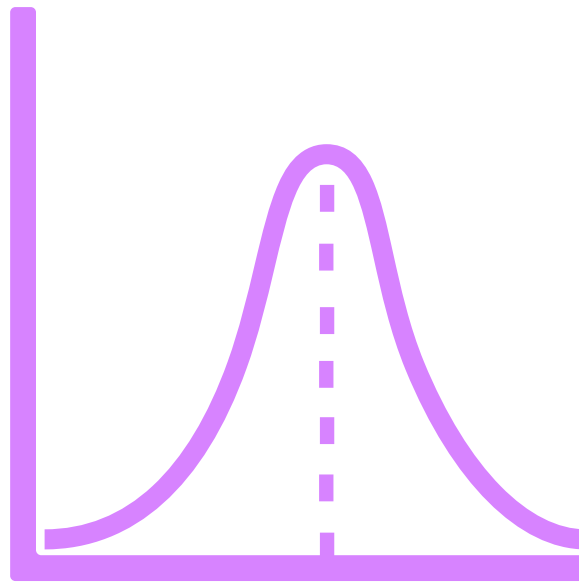
\mathcal{D}_3



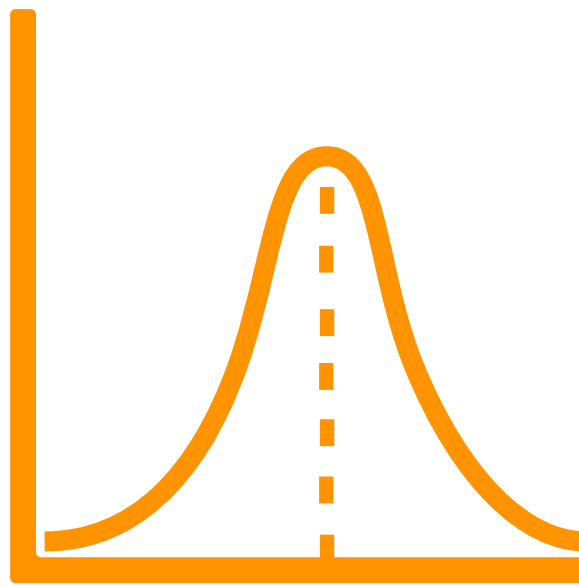
independent
(not necessarily
identical)

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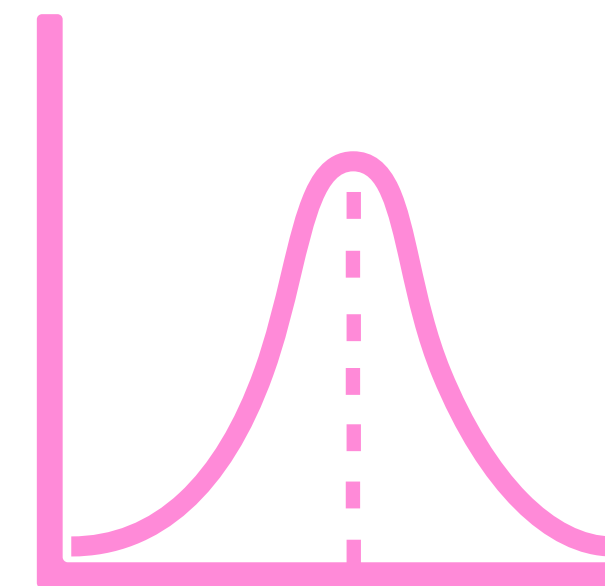
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\mathcal{D}_2



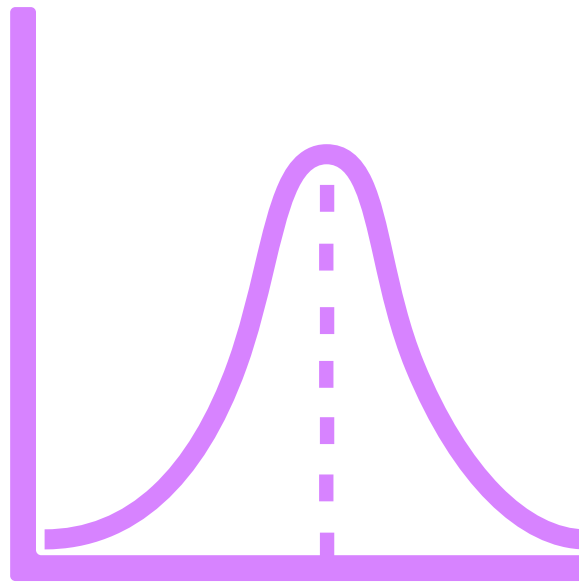
\mathcal{D}_3



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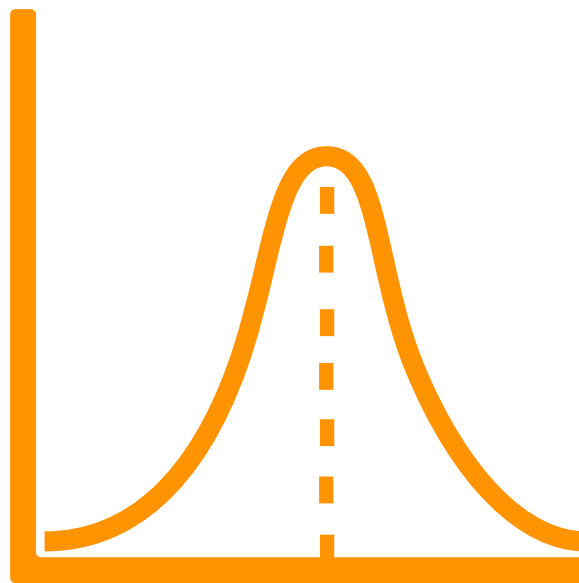
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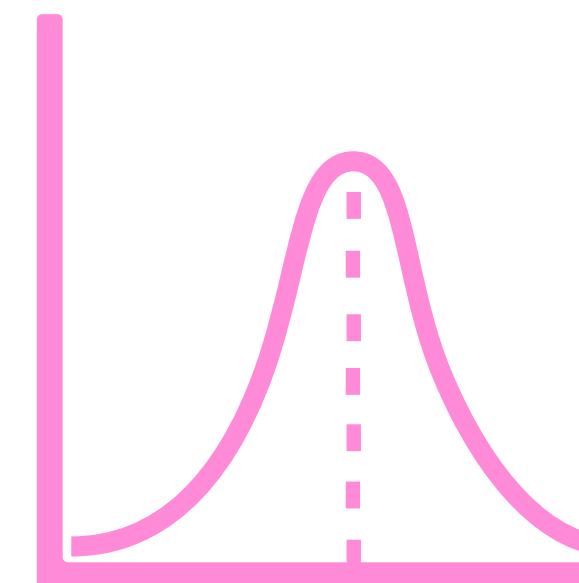
P_1

\mathcal{D}_2



P_2

\mathcal{D}_3



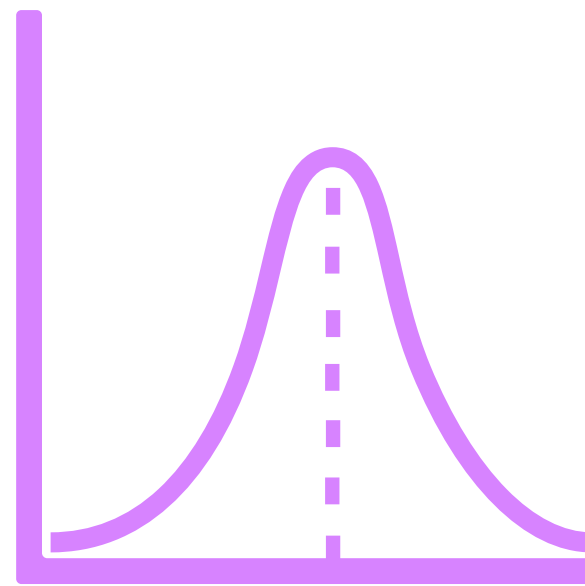
P_3

independent
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Problem Definition

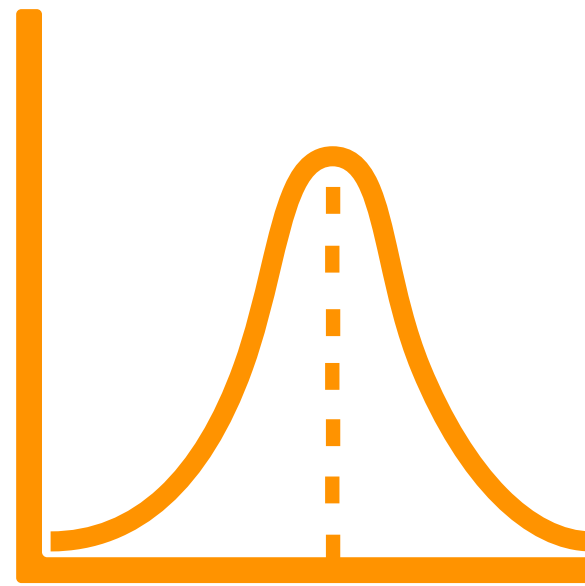
• Nonanticipatory

\mathcal{D}_1



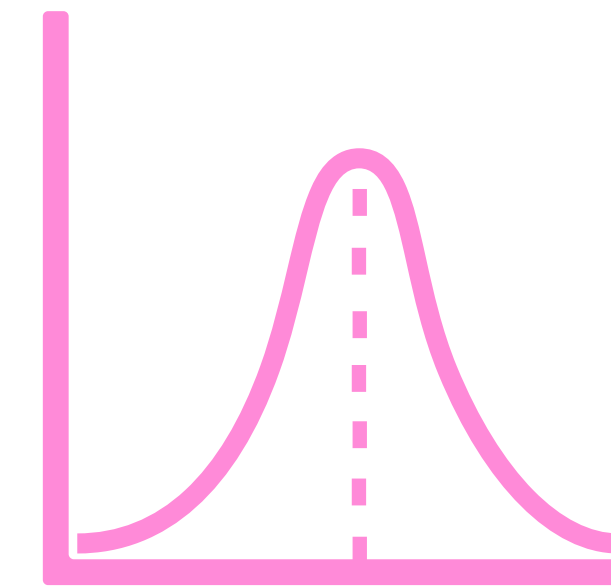
P_1

\mathcal{D}_2



P_2

\mathcal{D}_3

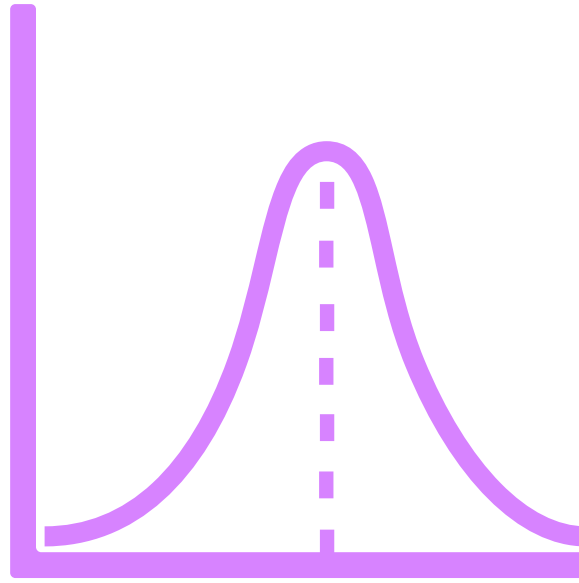


P_3

independent
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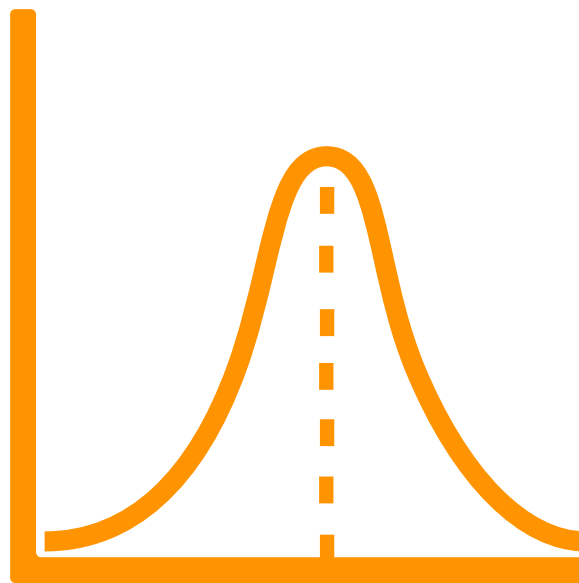
Problem Definition

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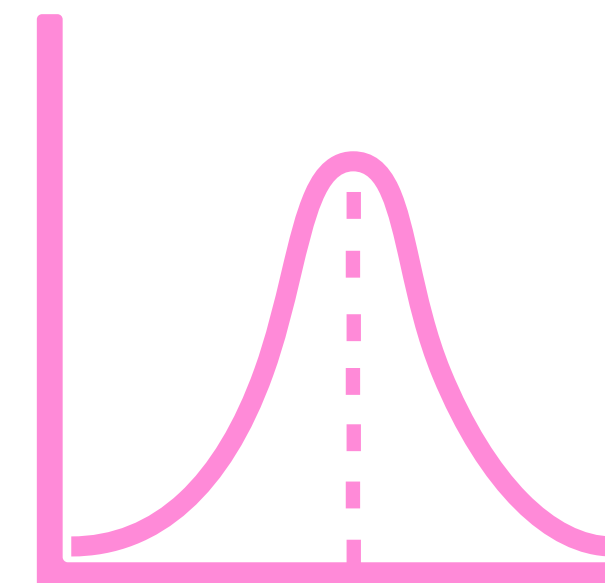
P_1

\mathcal{D}_2



P_2

\mathcal{D}_3

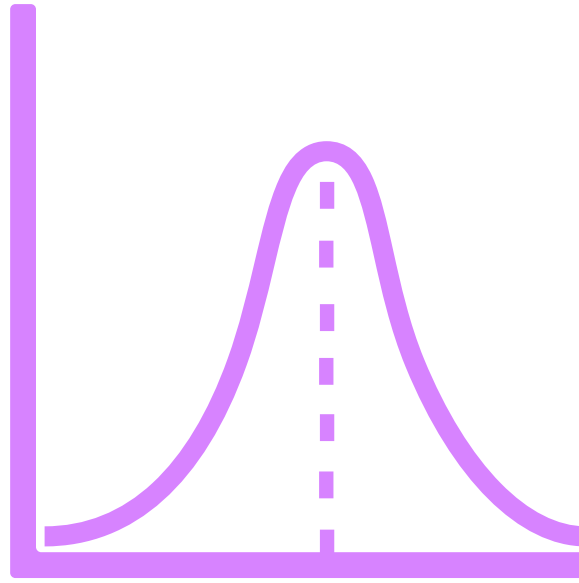


P_3

- Nonanticipatory
- Preemption allowed

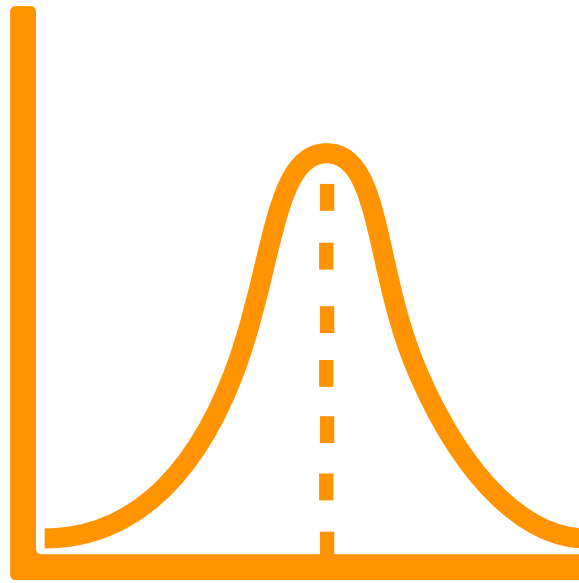
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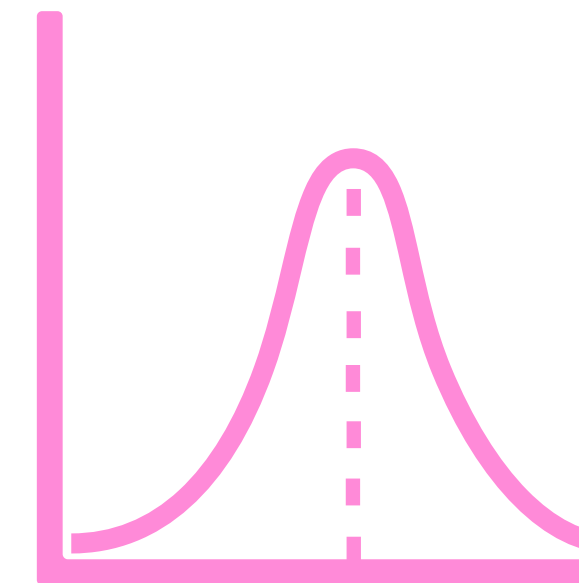
P_1

\mathcal{D}_2



P_2

\mathcal{D}_3

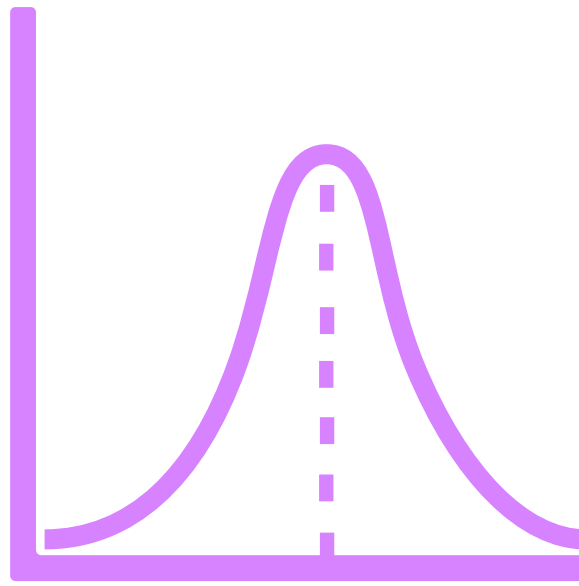


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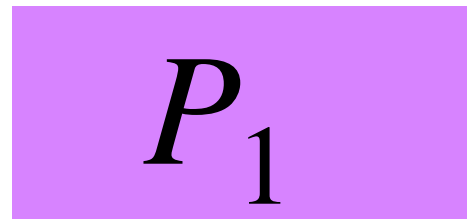
- Nonanticipatory
- Preemption allowed
- No release dates

independent
(not necessarily
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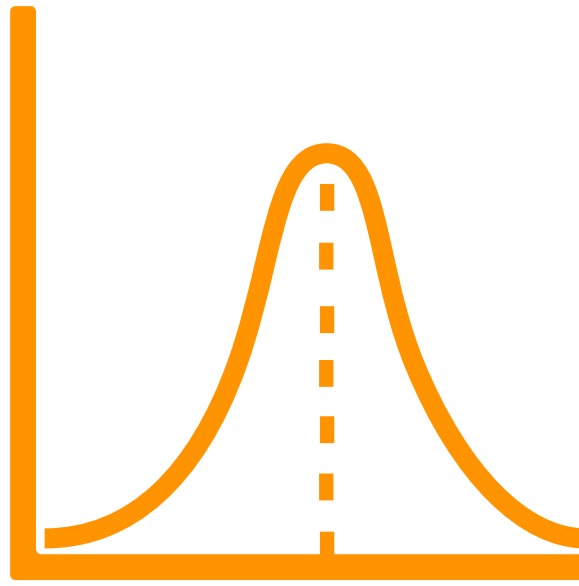
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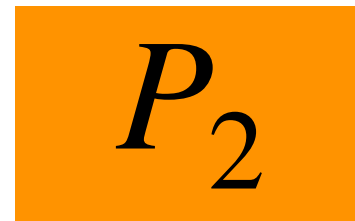
P_1



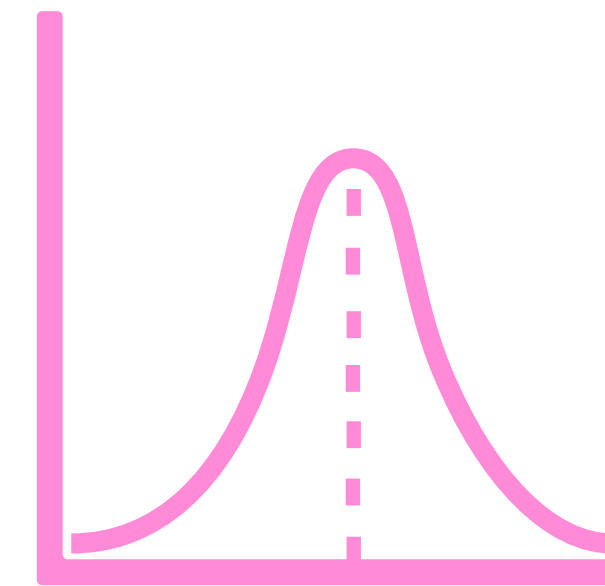
\mathcal{D}_2



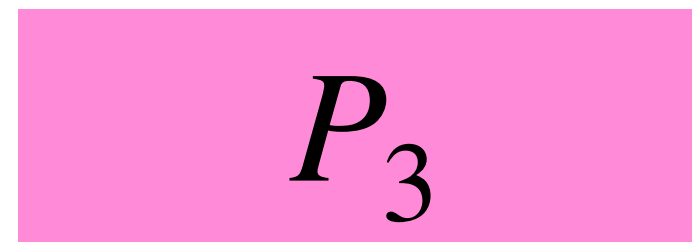
P_2



\mathcal{D}_3



P_3

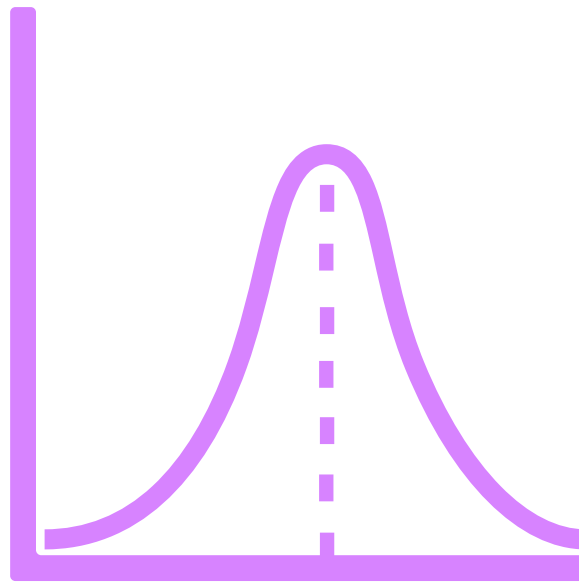


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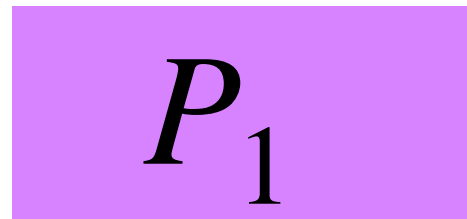
- Nonanticipatory
- Preemption allowed
- No release dates
- Single machine

independent
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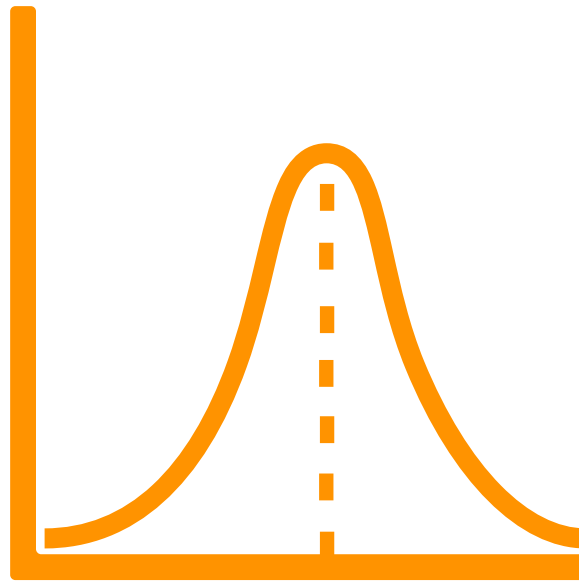
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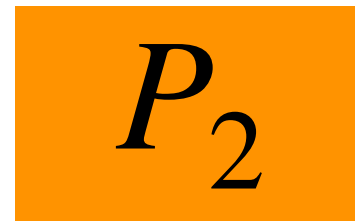
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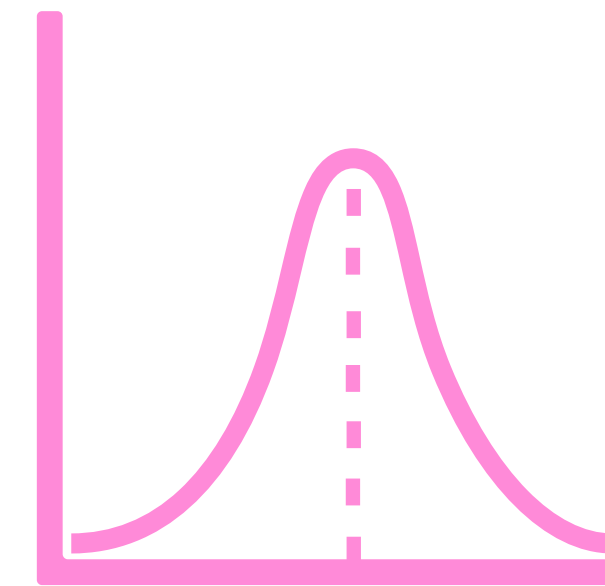
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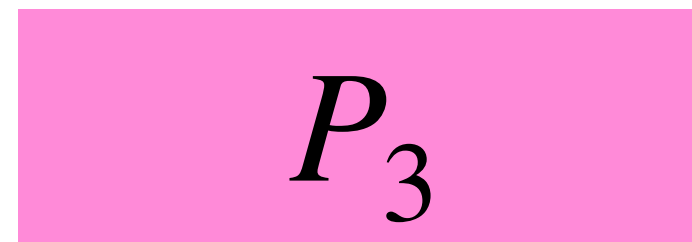
P_2



\mathcal{D}_3



P_3

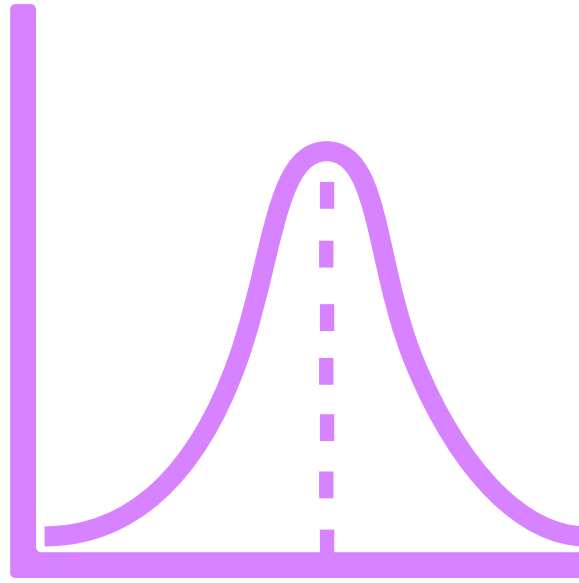


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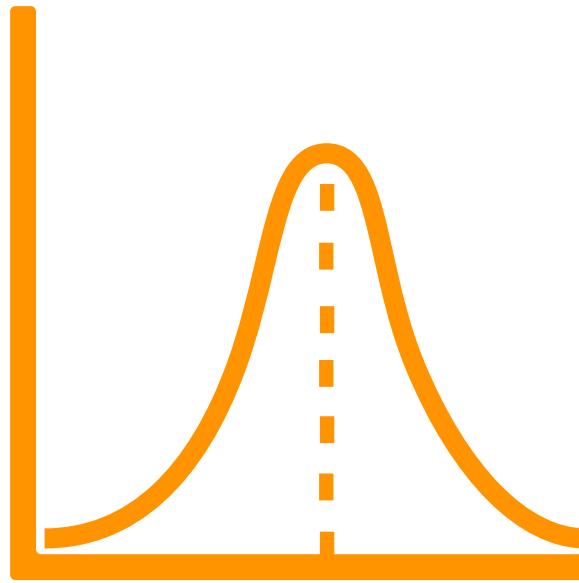
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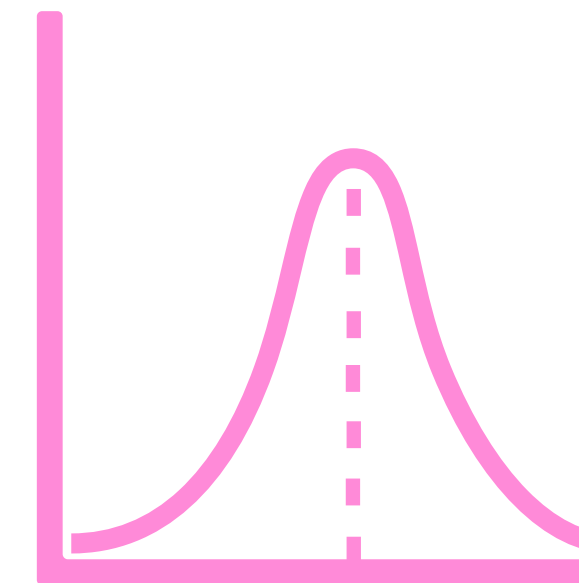
P_1

\mathcal{D}_2



P_2

\mathcal{D}_3

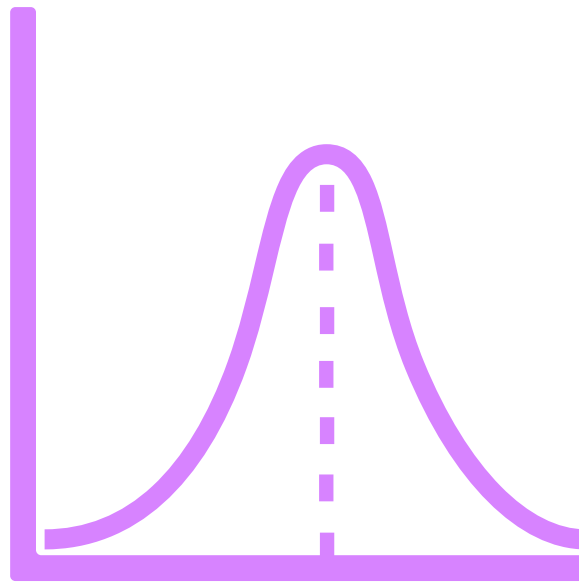


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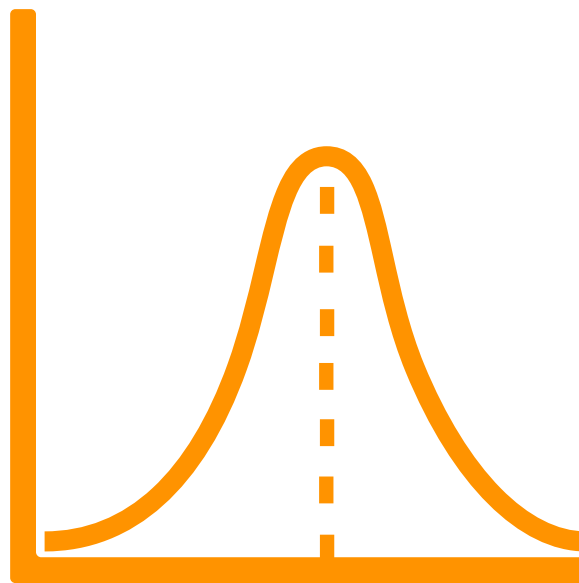
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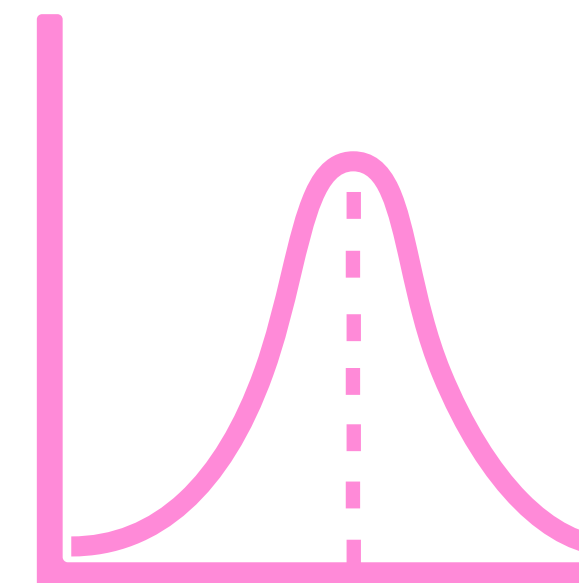
P_1

\mathcal{D}_2



P_2

\mathcal{D}_3



P_3

Problem Definition

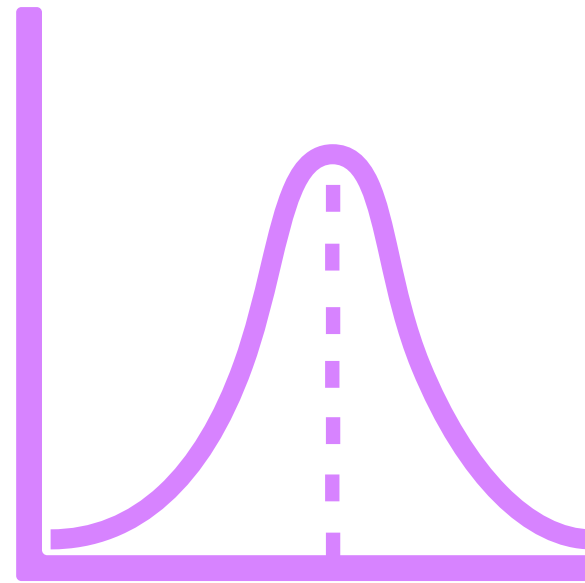
- Nonanticipatory
- Preemption allowed
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Goal: nonanticipatory policy minimizing

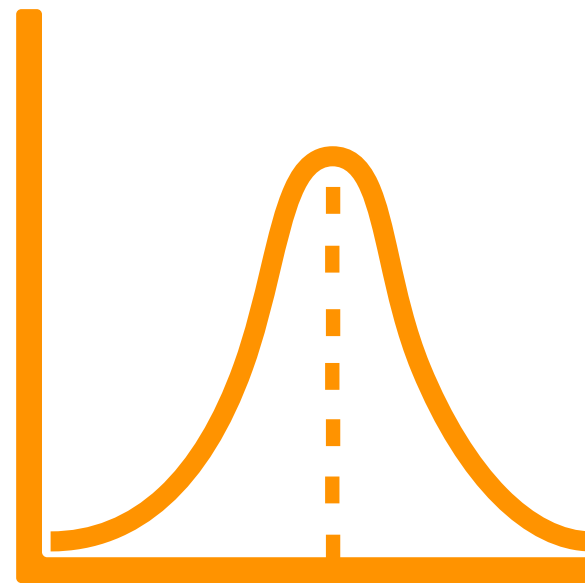
$$\mathbb{E} \left[\sum_{j=1}^n C_j \right]$$

independent
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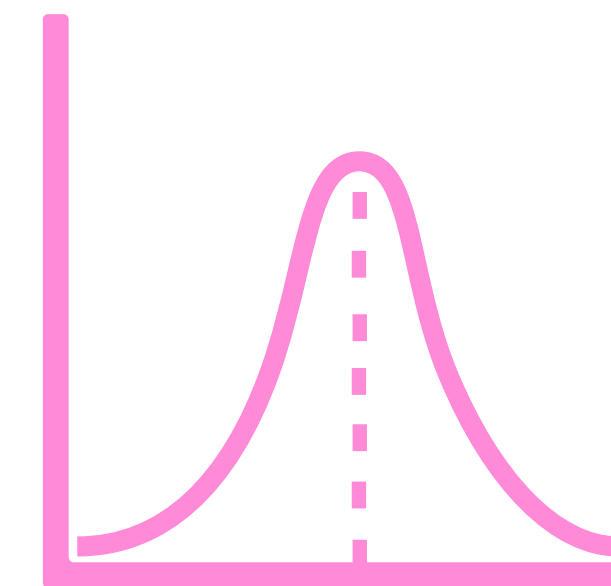
\mathcal{D}_1



\mathcal{D}_2



\mathcal{D}_3



Problem Definition

P_1

P_2

P_3

only learn size when
job completes

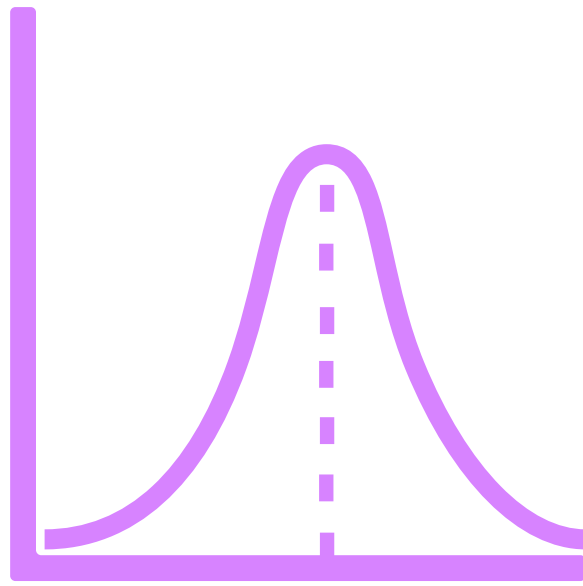
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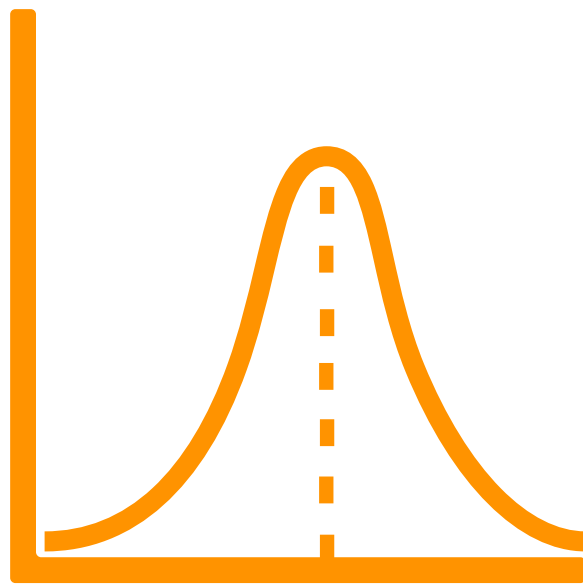
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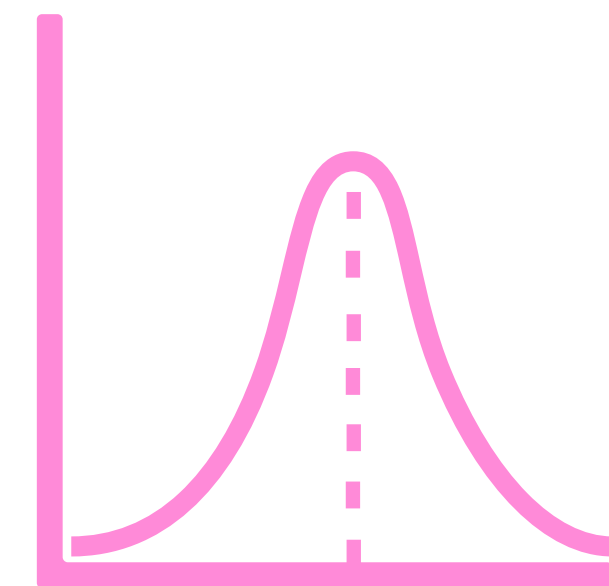
\mathcal{D}_1



\mathcal{D}_2



\mathcal{D}_3



Optimal Policies

P_1

P_2

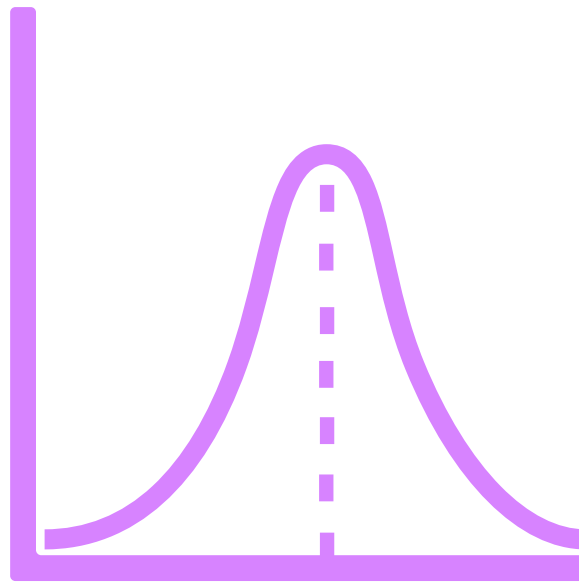
P_3

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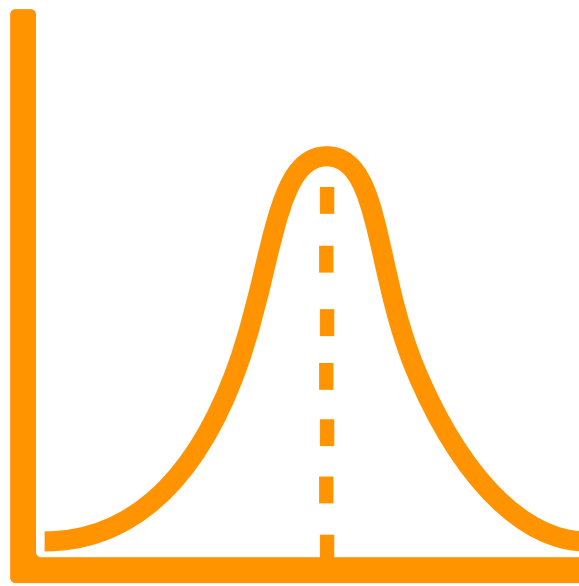
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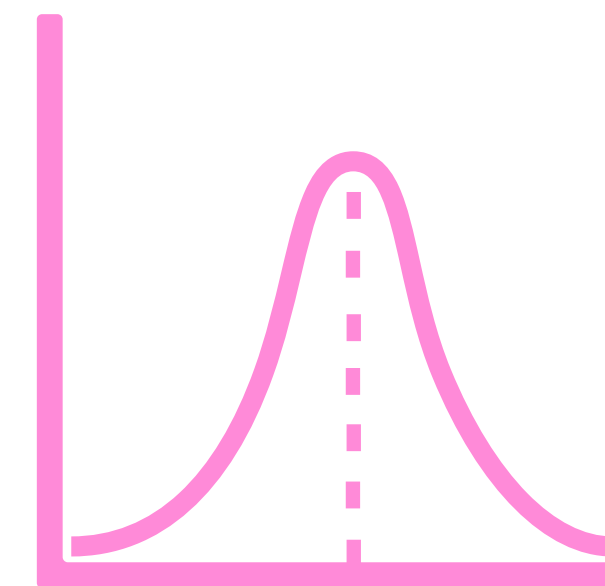
\mathcal{D}_1



\mathcal{D}_2



\mathcal{D}_3



Optimal Policies

P_1

P_2

P_3

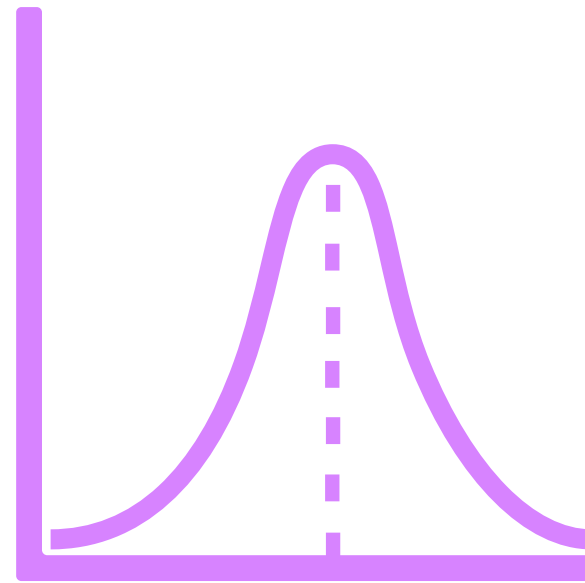
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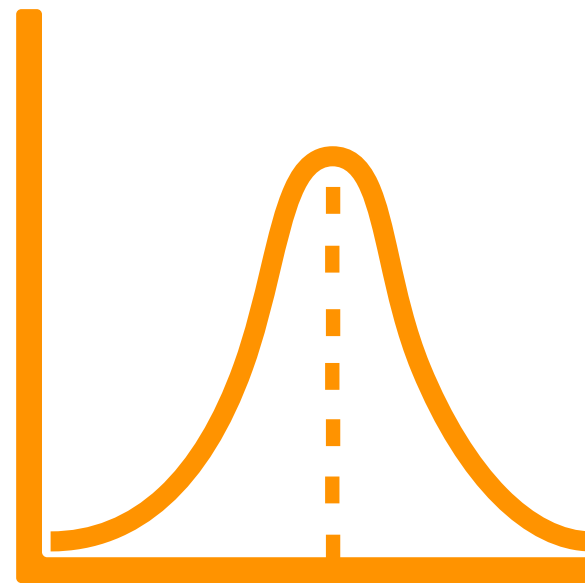
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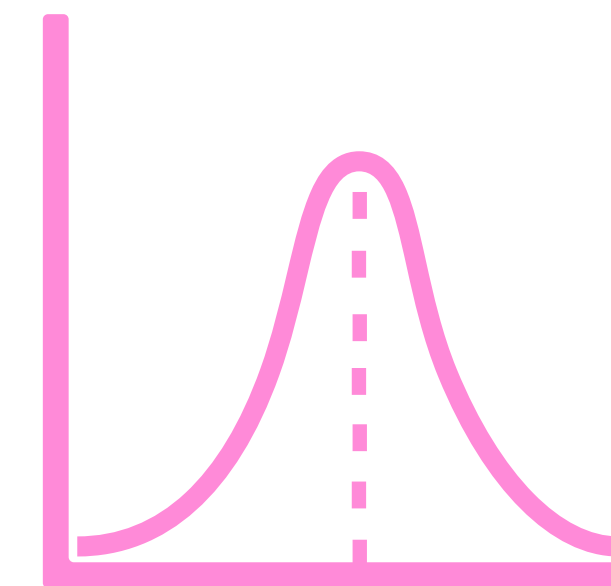
\mathcal{D}_1



\mathcal{D}_2



\mathcal{D}_3



Optimal Policies

P_1

P_2

P_3

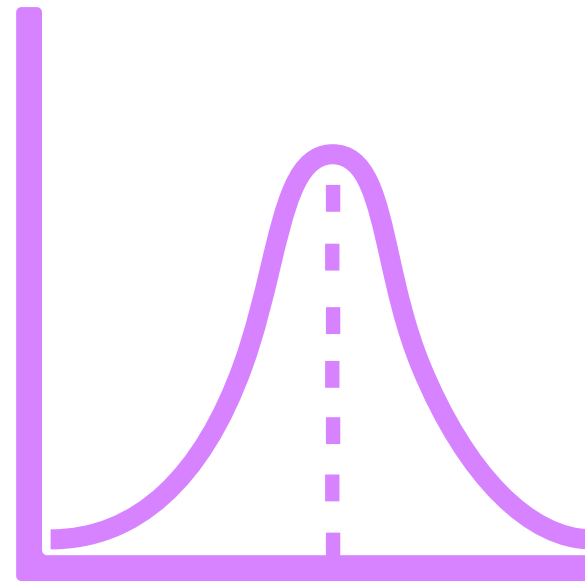
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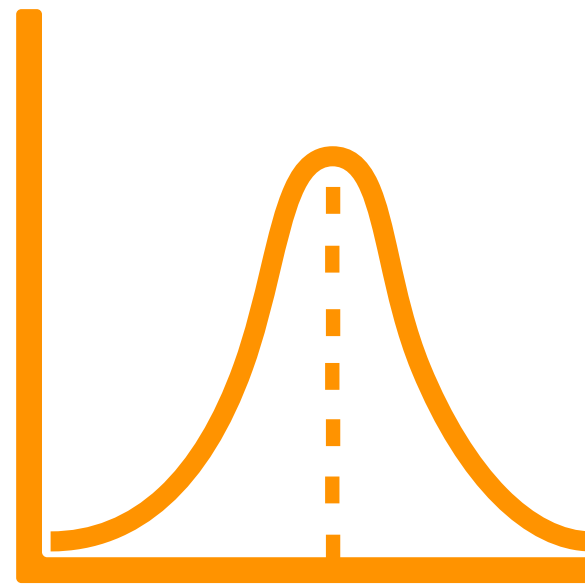
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- **Decreasing hazard rates:** SERPT

independent
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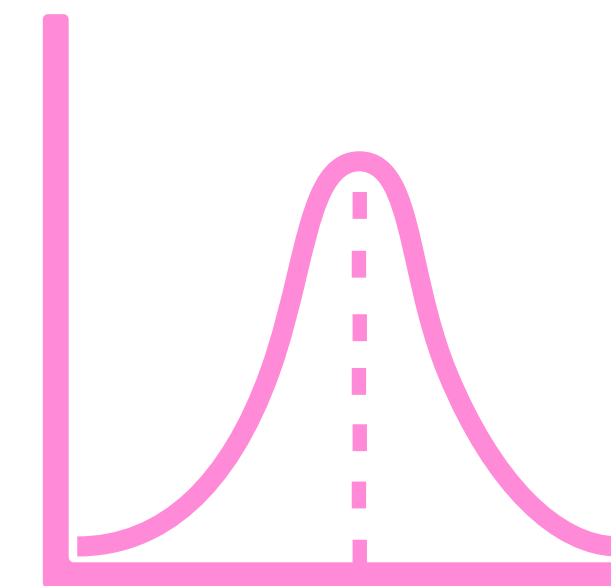
\mathcal{D}_1



\mathcal{D}_2



\mathcal{D}_3



Optimal Policies

P_1

P_2

P_3

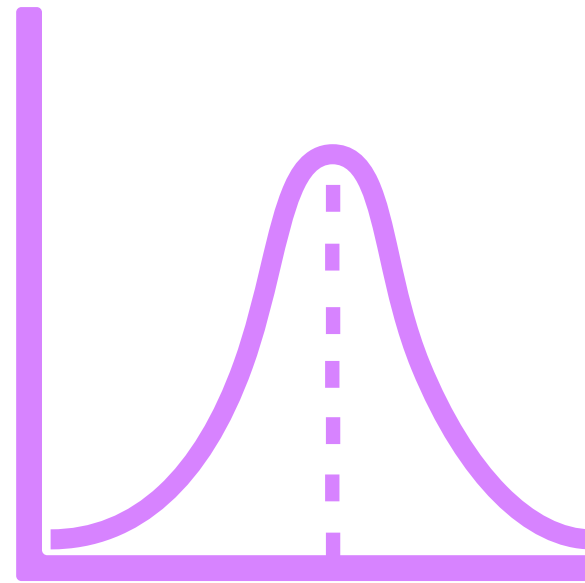
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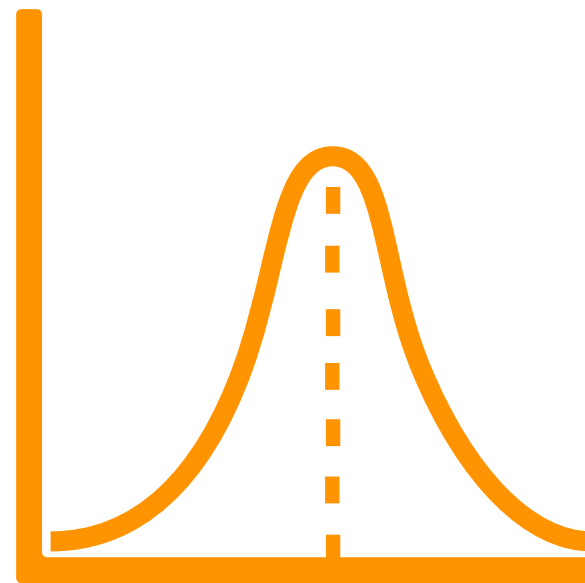
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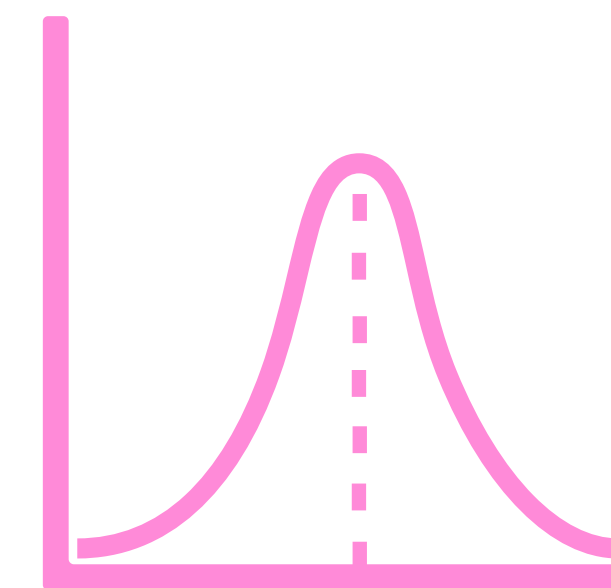
\mathcal{D}_1



\mathcal{D}_2



\mathcal{D}_3



Optimal Policies

P_1

P_2

P_3

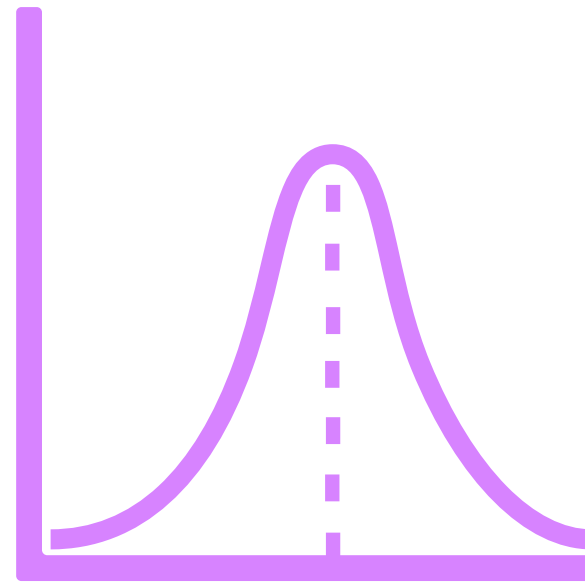
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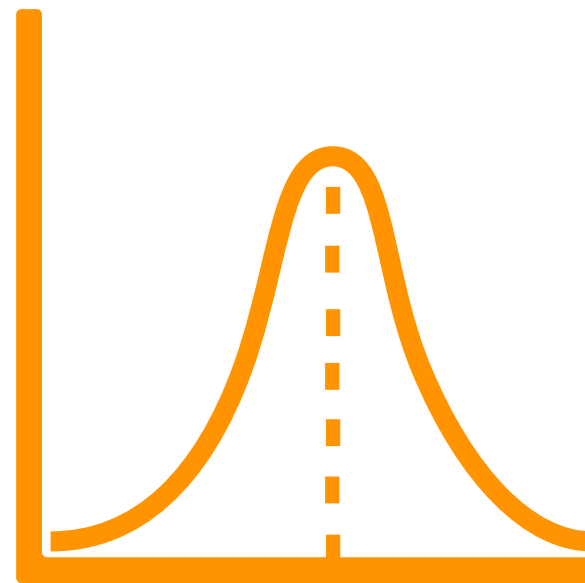
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- **In general:** Gittins Index Priority Policy

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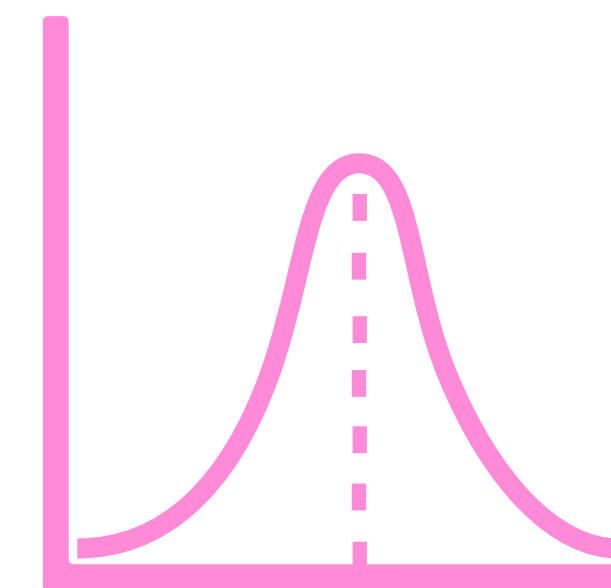
\mathcal{D}_1



\mathcal{D}_2



\mathcal{D}_3



Optimal Policies

P_1

P_2

P_3

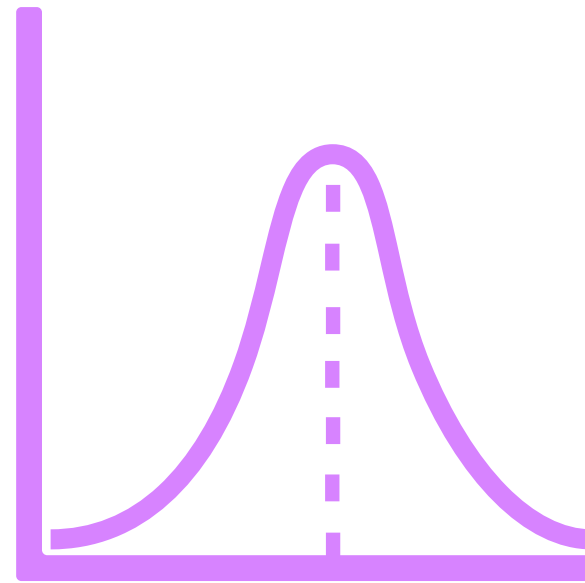
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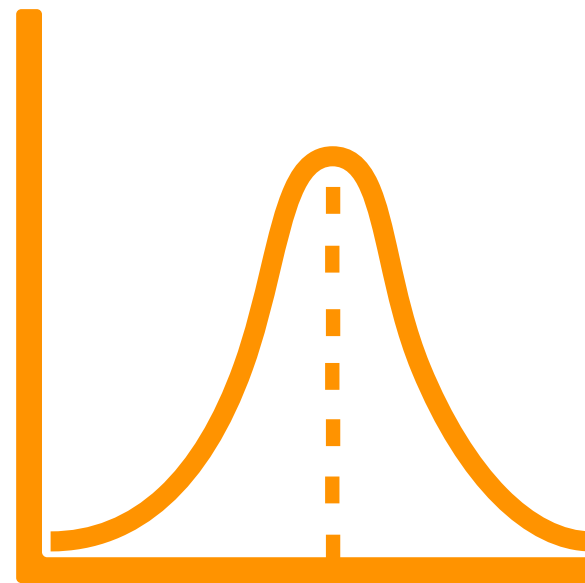
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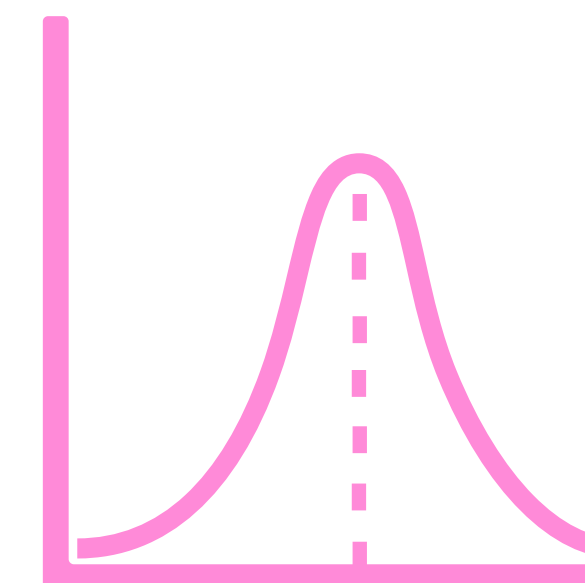
\mathcal{D}_1



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Optimal Policies

P_1

P_2

P_3

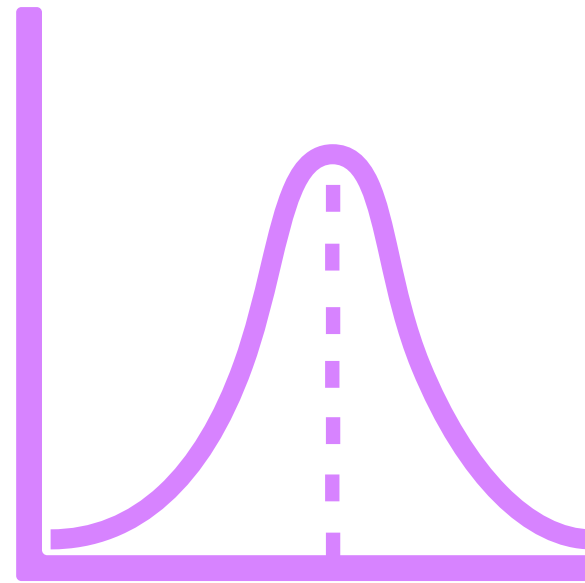
Goal: nonanticipatory policy minimizing

$$\mathbb{E} \left[\sum_{j=1}^n c_j \right]$$

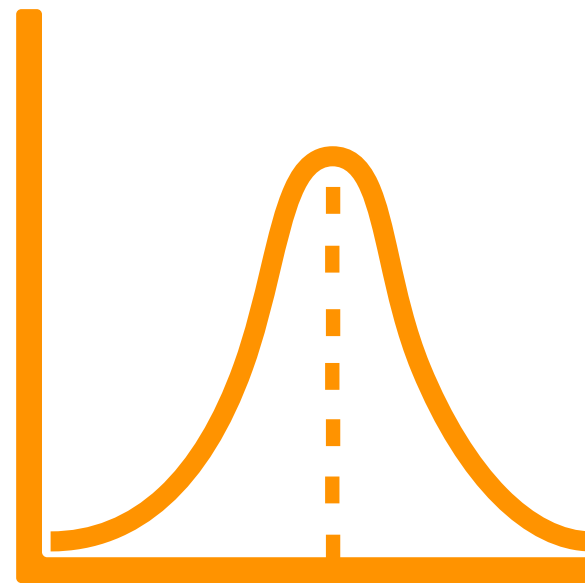
- **Deterministic:** SPT
- **Decreasing hazard rates:** SERPT
- **Increasing hazard rates:** SEPT
- **In general:** Gittins Index Priority Policy

independent
(not necessarily
identical)

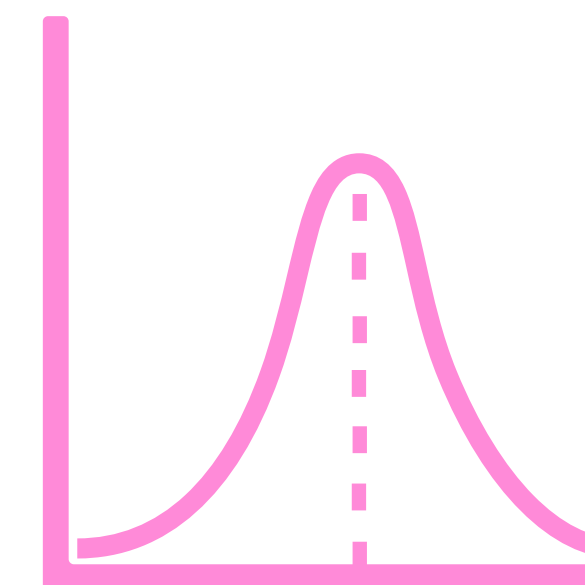
\mathcal{D}_1



\mathcal{D}_2



\mathcal{D}_3



Optimal Policies

P_1

P_2

P_3

Goal: nonanticipatory policy minimizing

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Policy Robustness

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Policy Robustness

Robust: SEPT

Why: cost expressed in terms of
unconditional expected values

$$\sum_{j=1}^n (n - j) \cdot \mathbb{E}[P_j]$$

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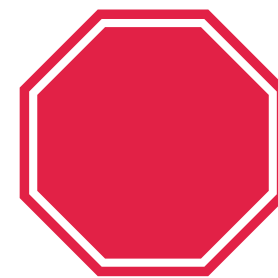
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Not Robust: Gittins



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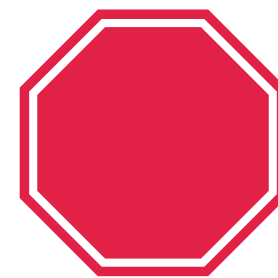
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Not Robust: Gittins



Why: policy based on conditional expectations and probabilities that are highly sensitive to error

Goal: nonanticipatory policy minimizing

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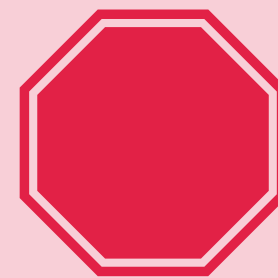
Gittins Index Policy

Robust: SEPT

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Not Robust: Gittins



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Gittins Index Policy

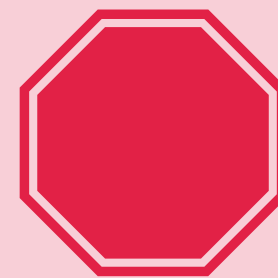
finite
support

Robust: SEPT

Why: cost expressed in terms of
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Not Robust: Gittins



Why: policy based on conditional
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Gittins Index Policy

finite
support

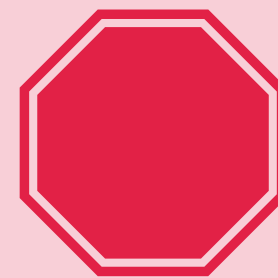
maximum support length

Robust: SEPT

Why: cost expressed in terms of
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Not Robust: Gittins



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Gittins Index Policy

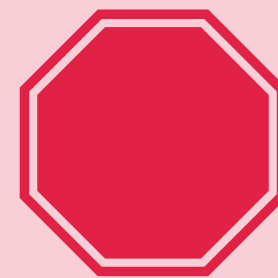
finite
support

Robust: SEPT

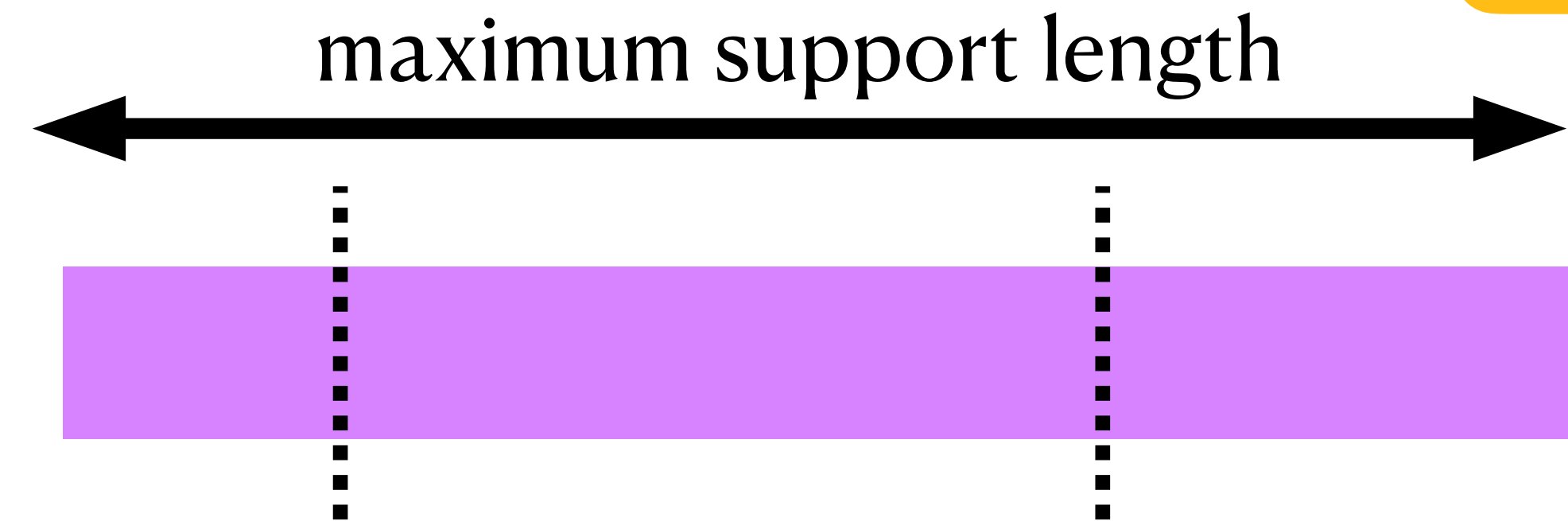
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Not Robust: Gittins



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Gittins Index Policy

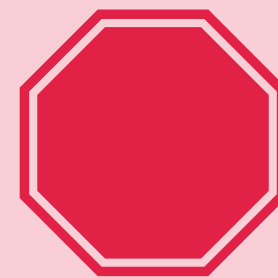
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Not Robust: Gittins



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maximum support length



Gittins Index Policy

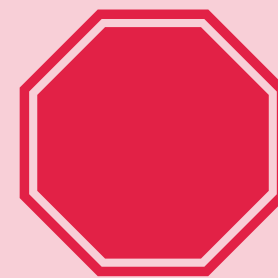
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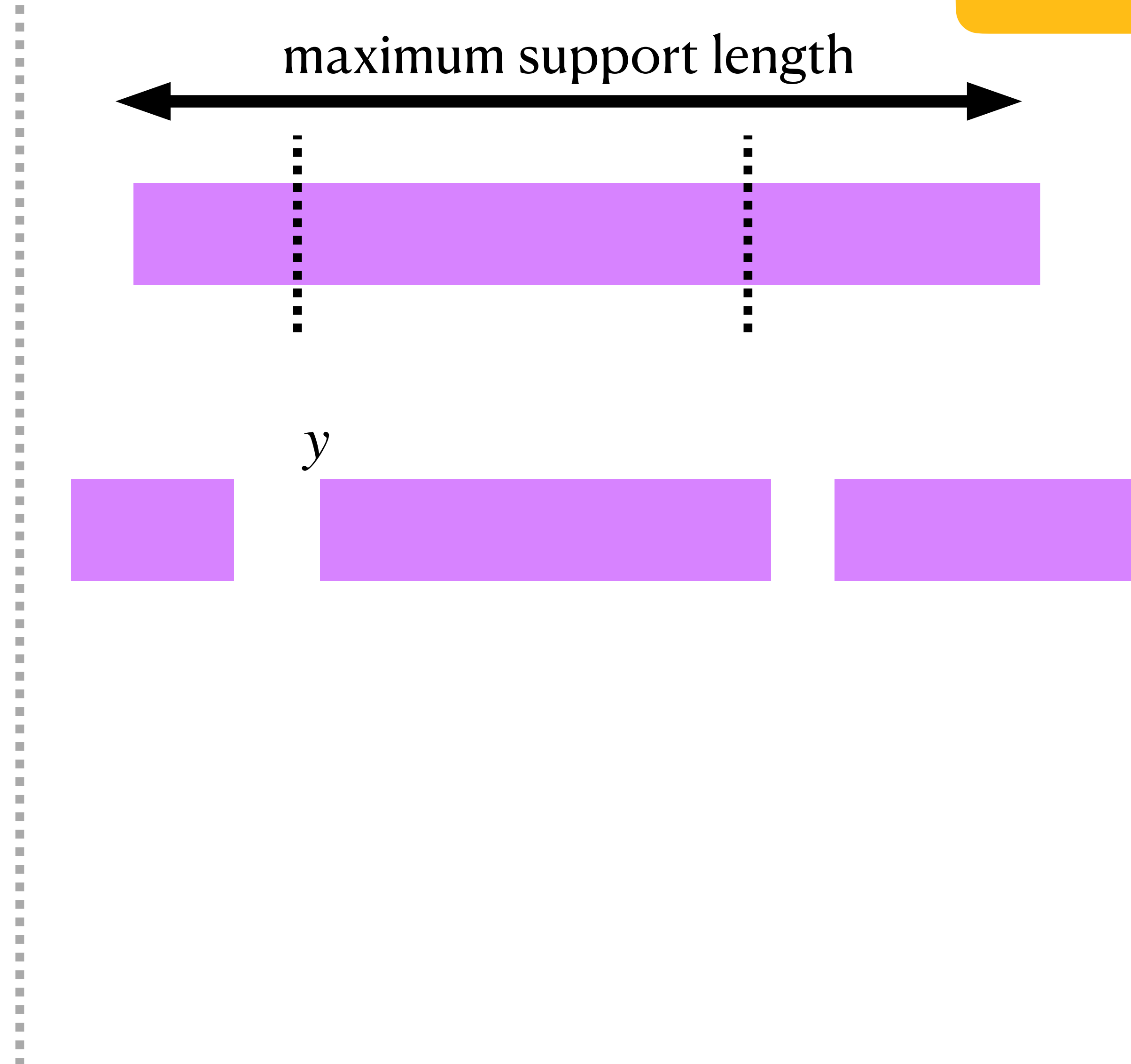
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Not Robust: Gittins



Why: policy based on conditional
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Gittins Index Policy

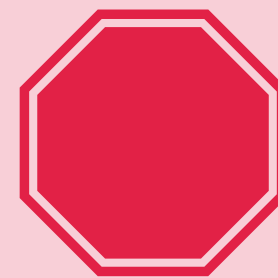
finite
support

Robust: SEPT

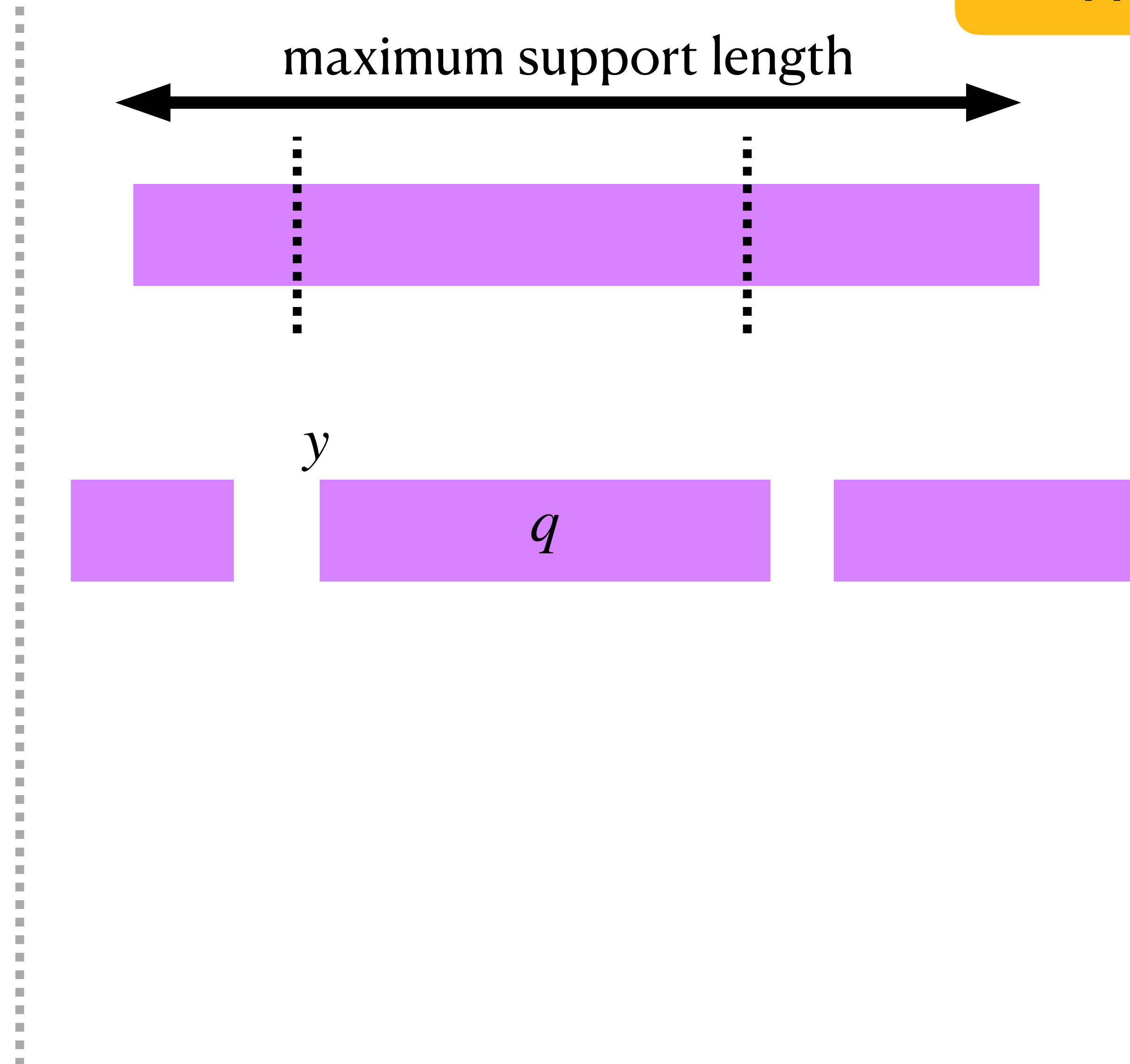
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Gittins Index Policy

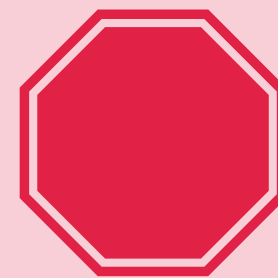
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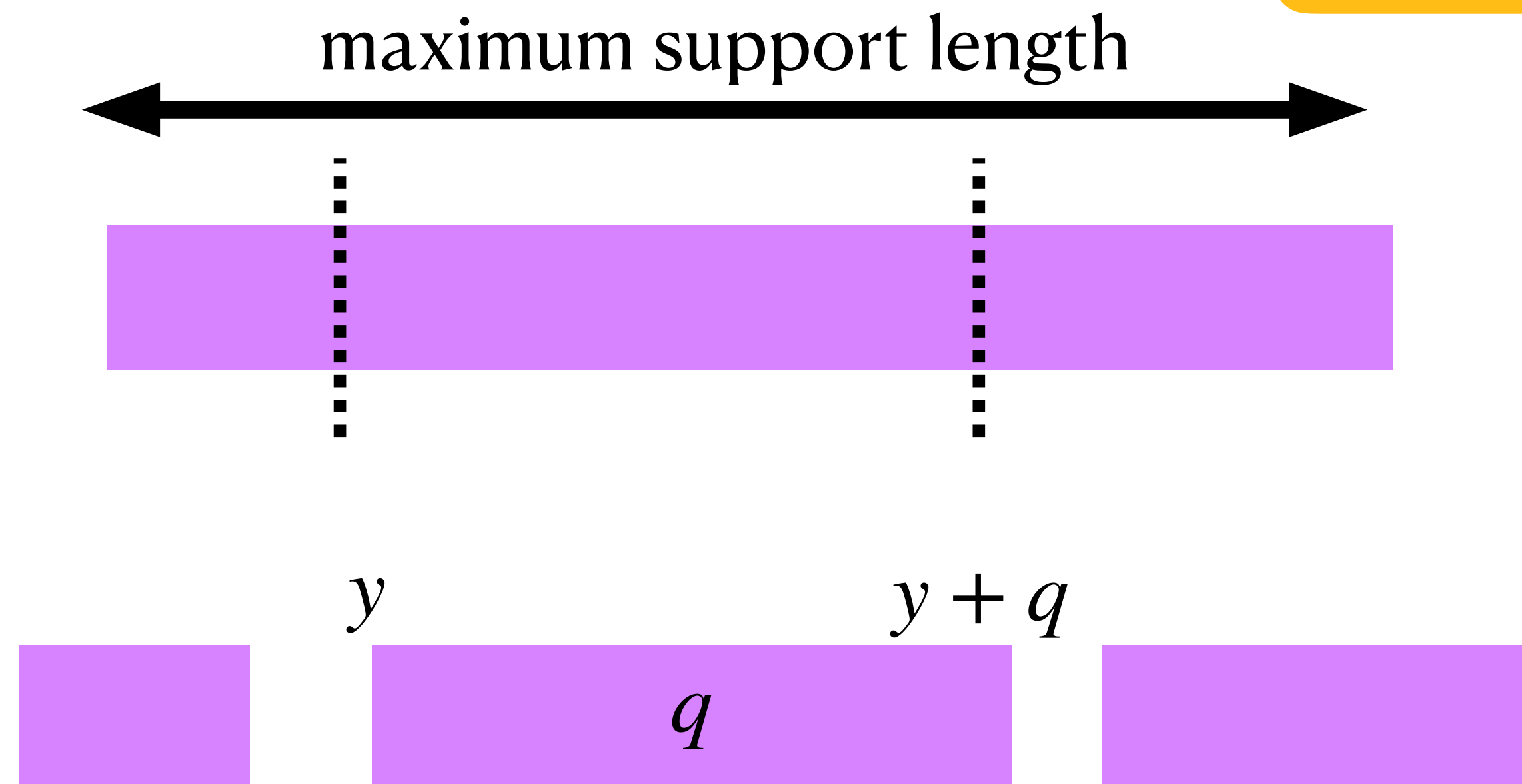
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Gittins Index Policy

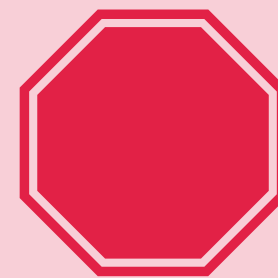
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support

Robust: SEPT

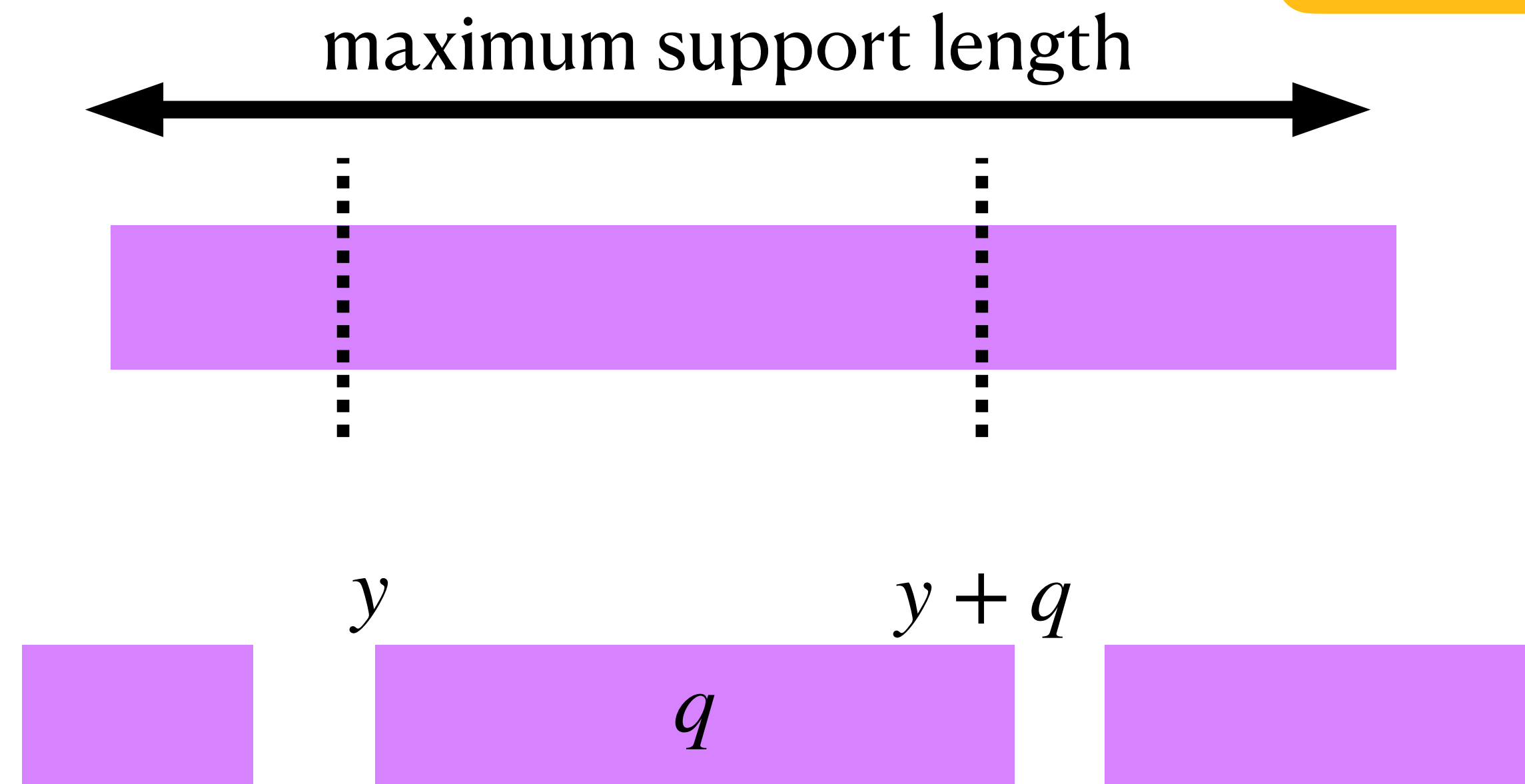
Why: cost expressed in terms of
unconditional expected values

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Not Robust: Gittins



Why: policy based on conditional
expectations and probabilities that
are highly sensitive to error



$$\max_{q \geq 0} \frac{\mathbb{P}(P_j - y \leq q \mid P_j > y)}{\mathbb{E}[\min\{P_j - y, q\} \mid P_j > y]}$$

Gittins Index Policy

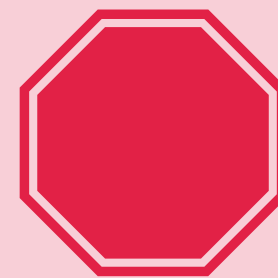
finite
support

Robust: SEPT

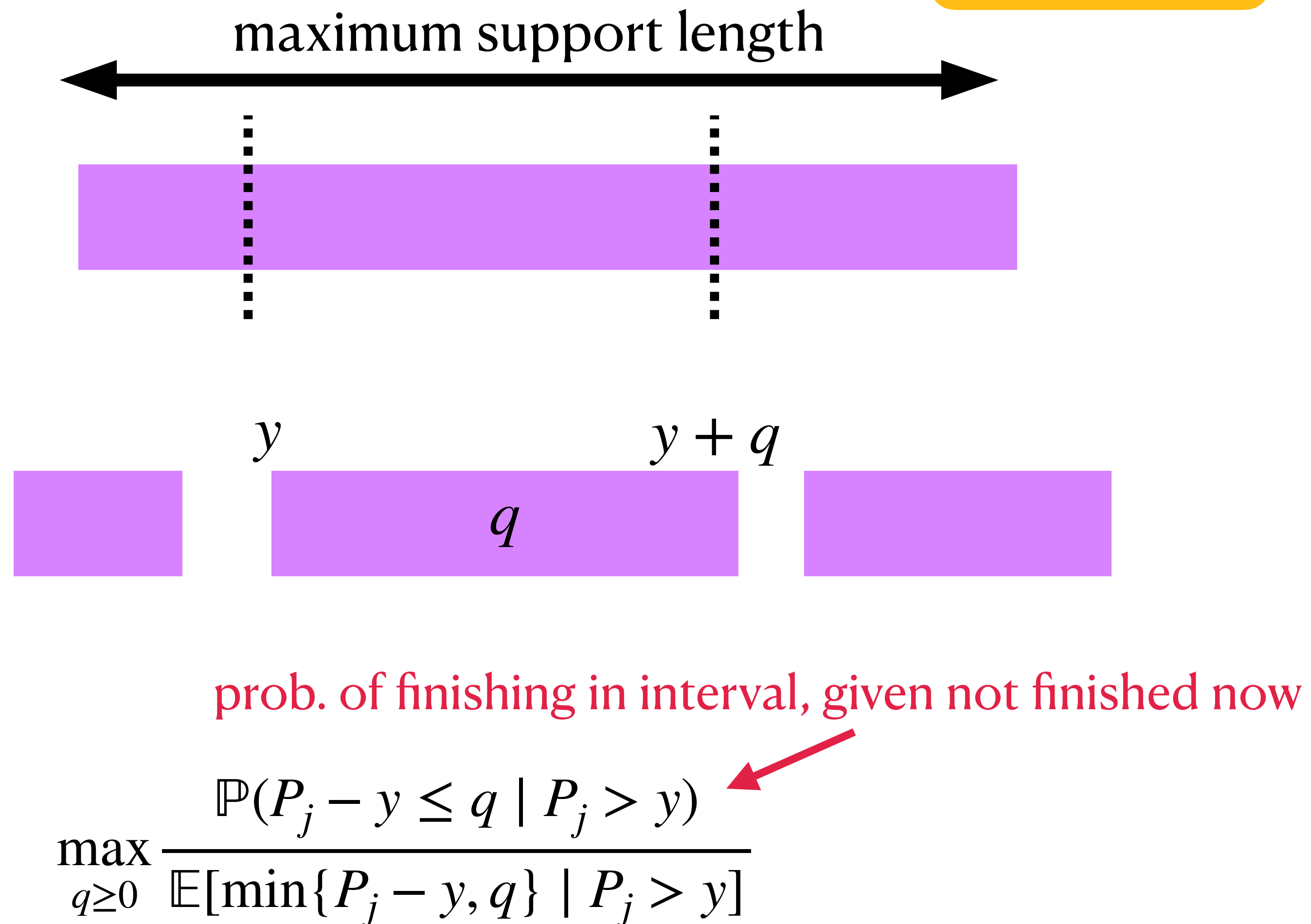
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Gittins Index Policy

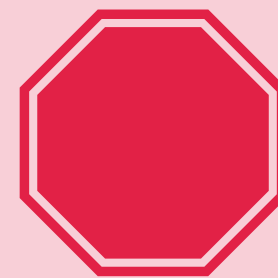
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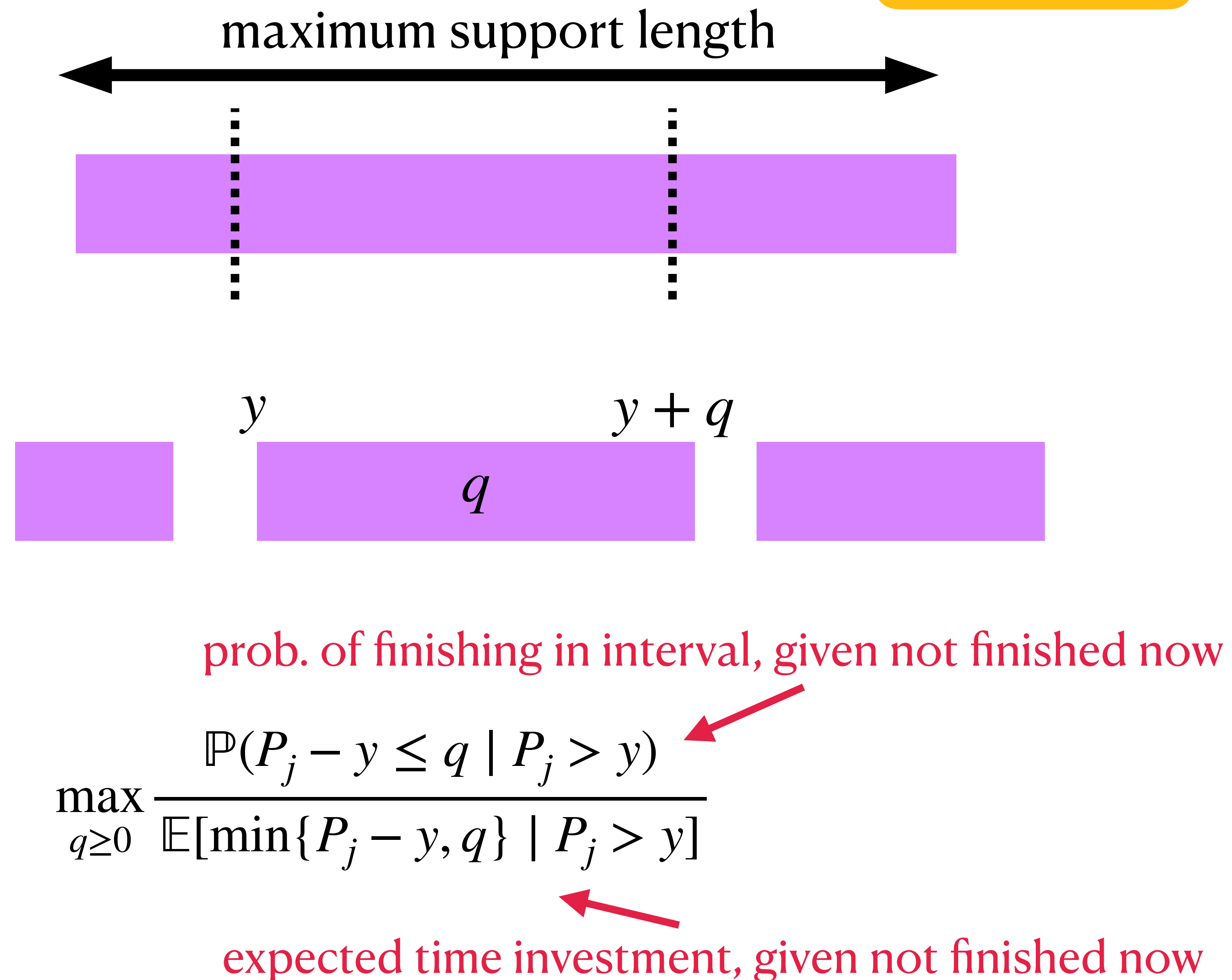
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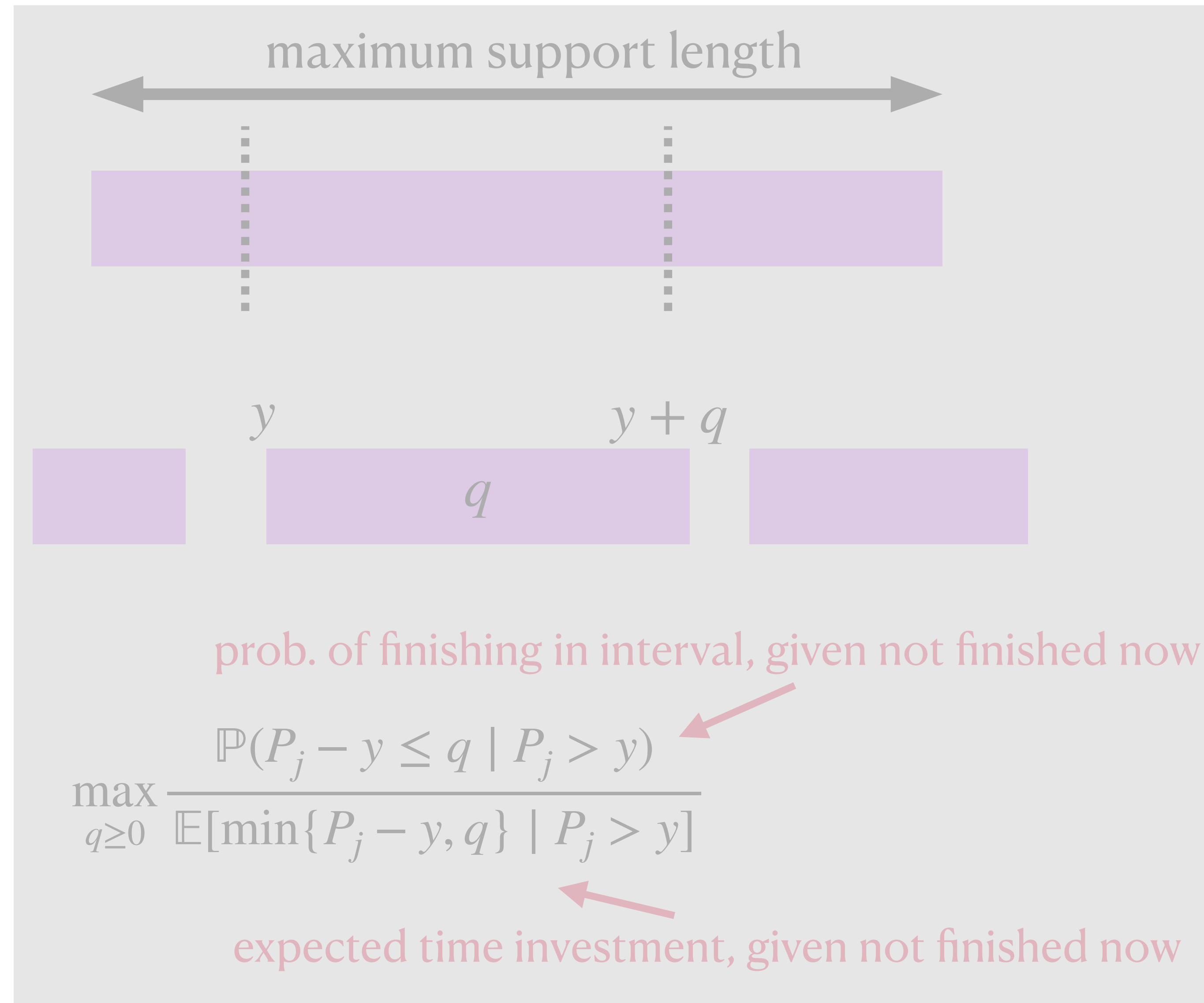
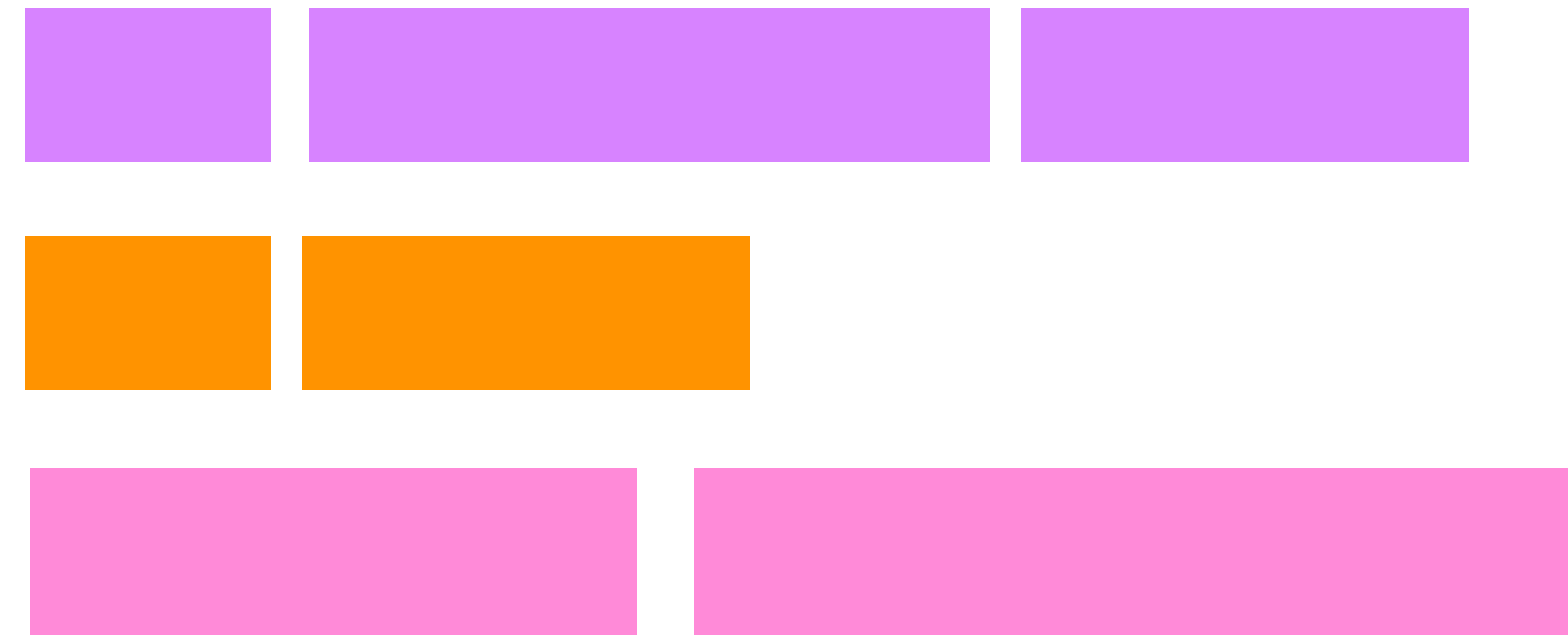
Not Robust: Gittins



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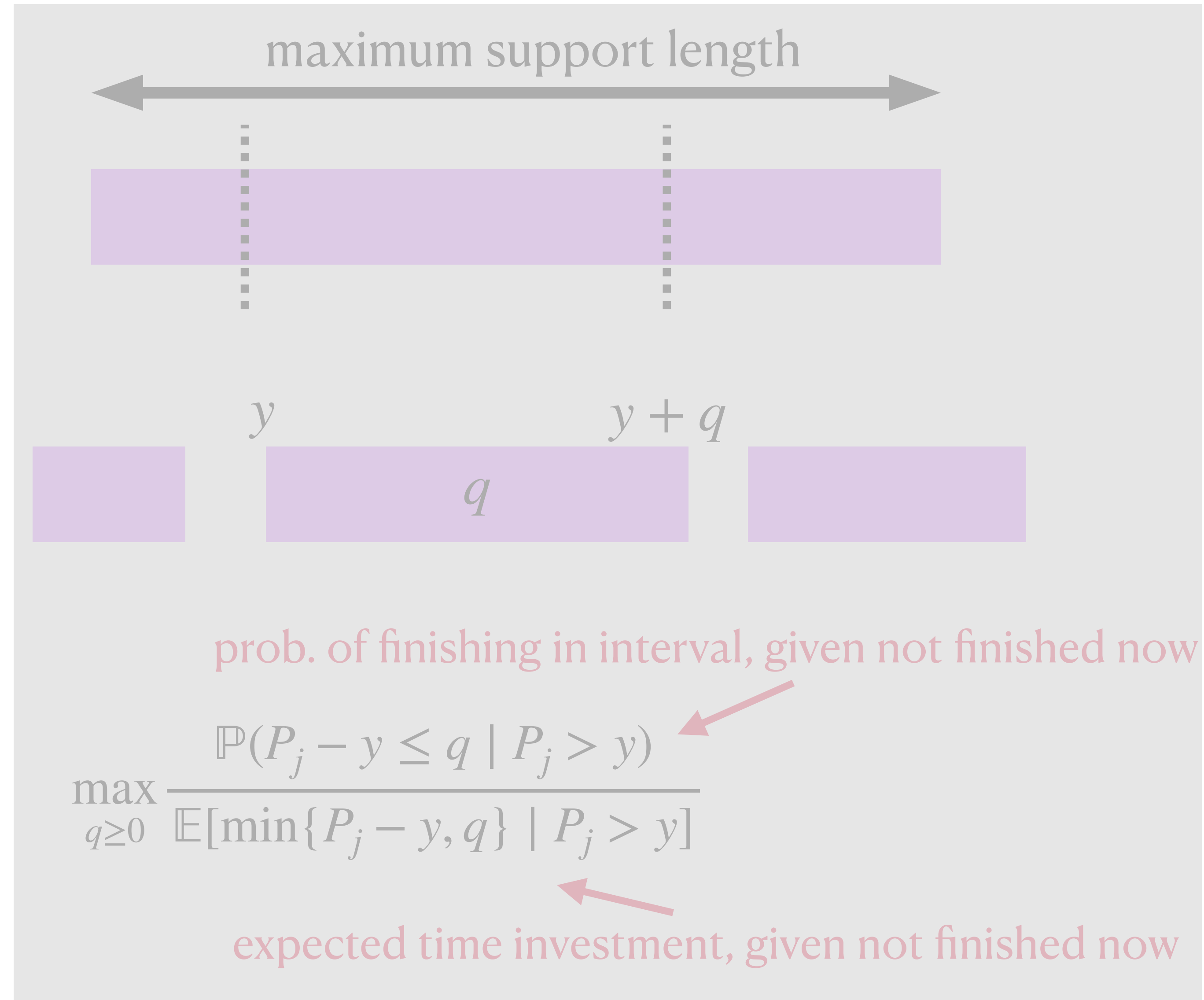


Gittins Index Policy



Gittins Index Policy

run in
decreasing
order of
rank



Gittins Index Policy

run in
decreasing
order of
rank



maximum support length

“Quanta” lengths AND ranks computed
independently for each job

y $y + q$
 q

prob. of finishing in interval, given not finished now

$$\max_{q \geq 0} \frac{\mathbb{P}(P_j - y \leq q \mid P_j > y)}{\mathbb{E}[\min\{P_j - y, q\} \mid P_j > y]}$$

expected time investment, given not finished now

Gittins Index Policy

run in
decreasing
order of
rank



maximum support length

“Quanta” lengths AND ranks computed
independently for each job

Quanta lengths **pre-computed** (i.e.,
not adaptive to job realizations)

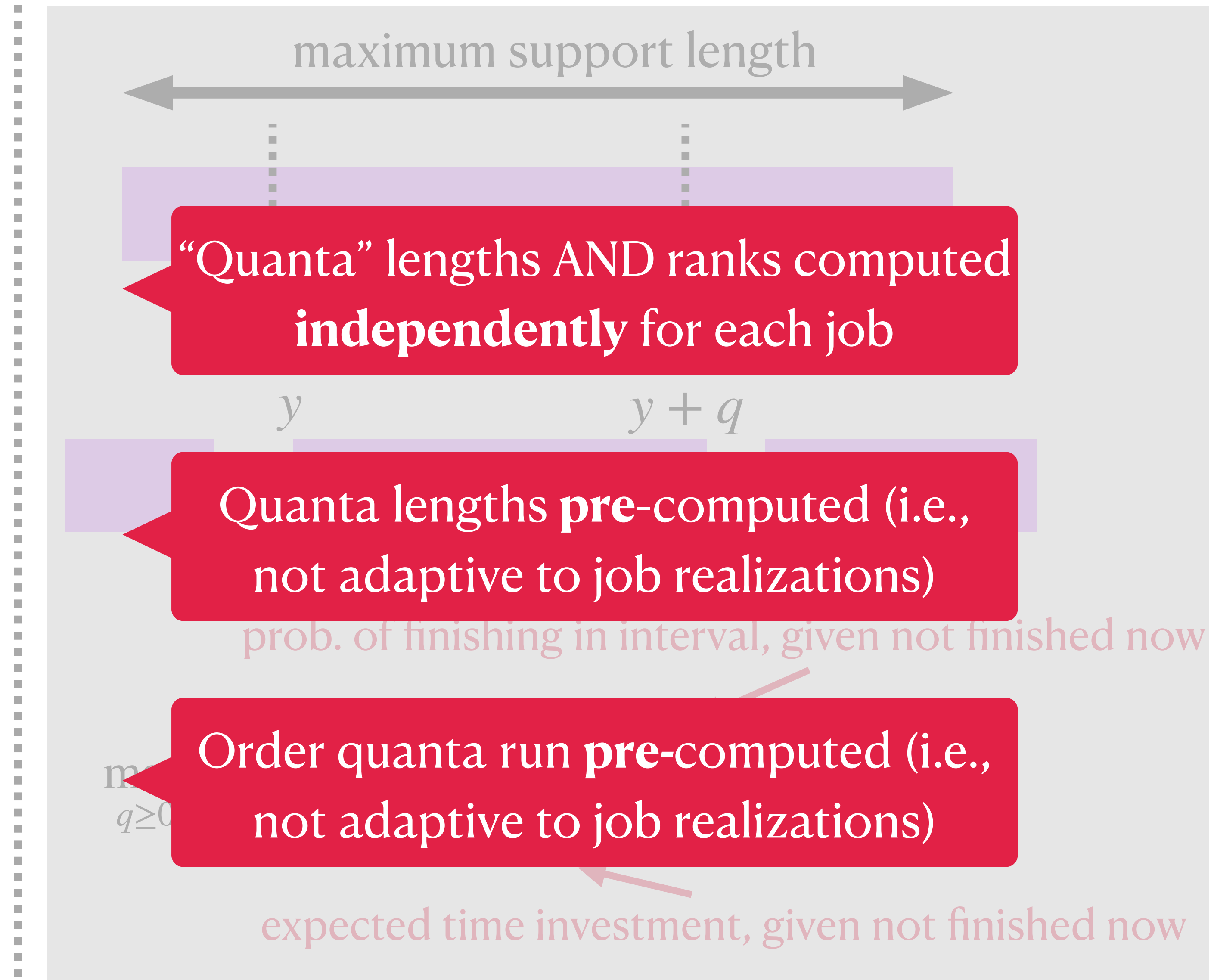
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expected time investment, given not finished now

Gittins Index Policy

run in
decreasing
order of
rank



Gittins is Not Robust

Gittins is Not Robust

Theorem. GIPP has arbitrarily poor performance even when the true distributions $\mathcal{J}^* = \{\mathcal{D}_j^*\}_{j=1}^n$ are arbitrarily “close” to the predicted distributions $\hat{\mathcal{J}} = \{\hat{\mathcal{D}}_j\}_{j=1}^n$

$$\text{GIPP}(\mathcal{J}^*, \hat{\mathcal{J}}) = \Omega(n) \cdot \text{GIPP}(\mathcal{J}^*, \mathcal{J}^*) = \text{OPT}(\mathcal{J}^*)^\dagger$$

Cost of Gittins on true distributions, given predicted distributions (to construct quanta, etc.)

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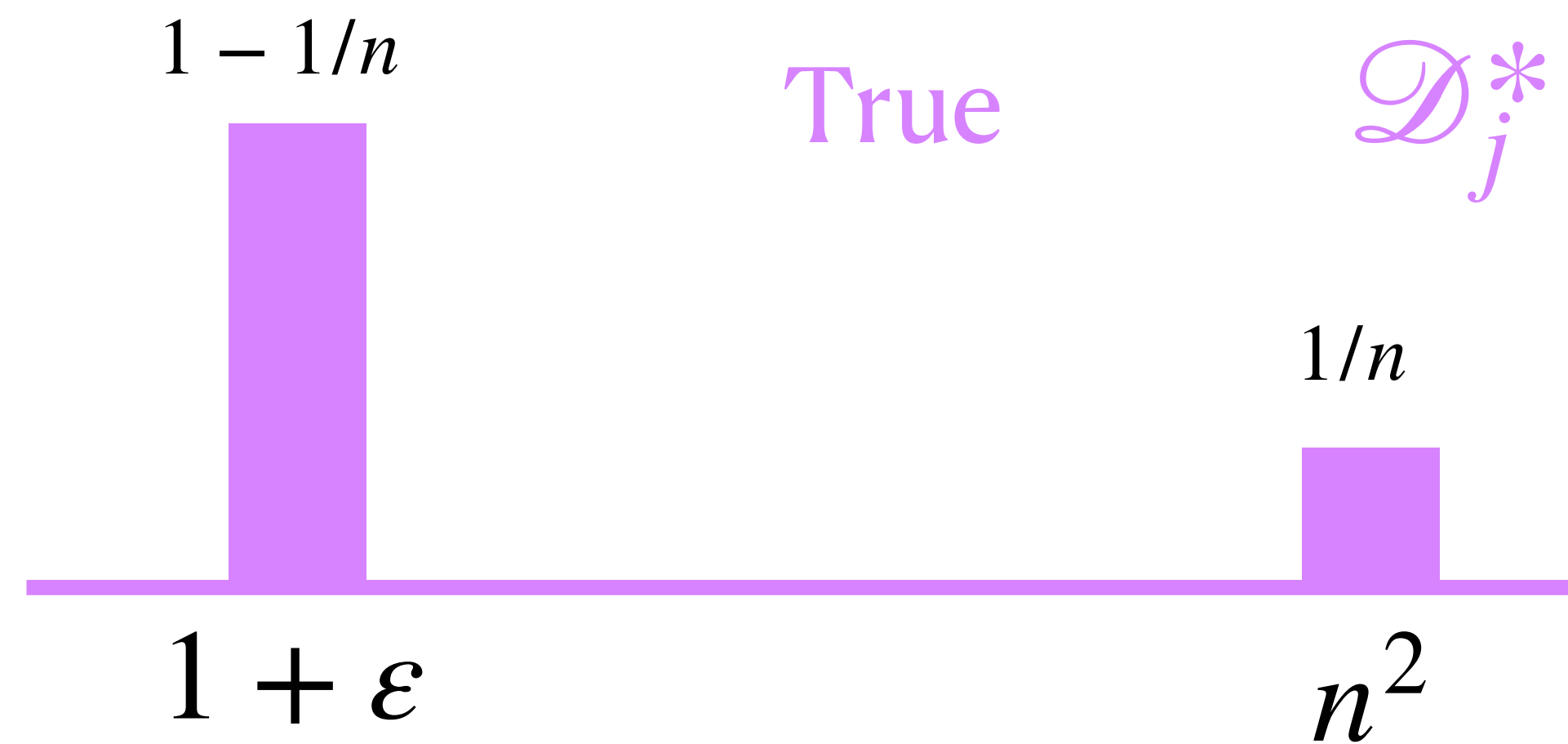
[†]Restricting to instances where $\text{GIPP}(\mathcal{J}^*, \hat{\mathcal{J}})$ completes all jobs of \mathcal{J}^*

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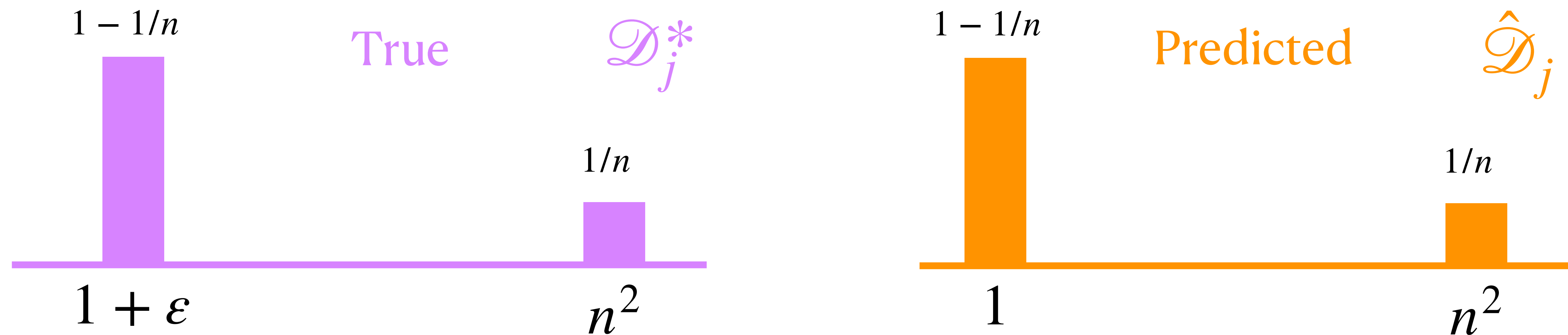
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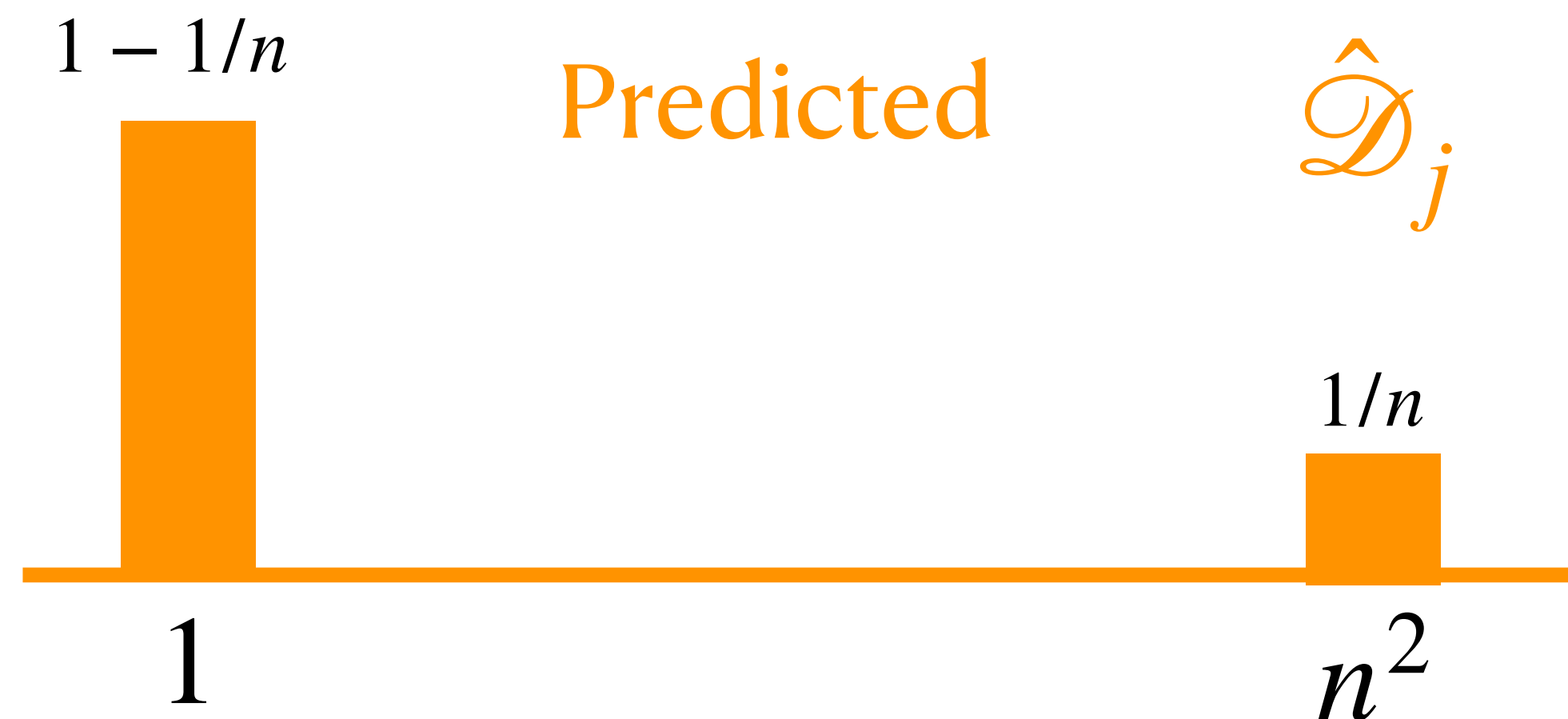
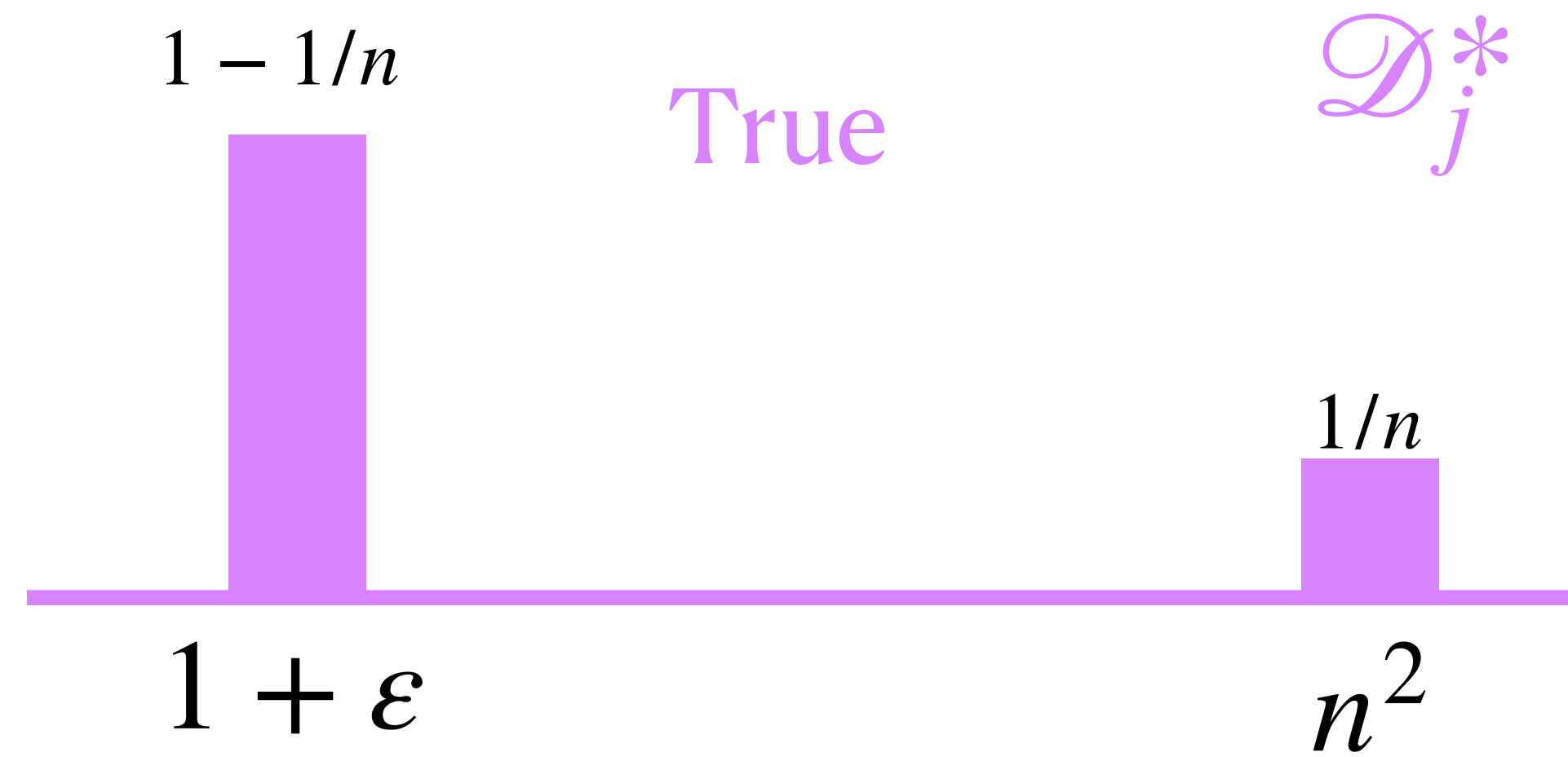


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Gittins is Not Robust

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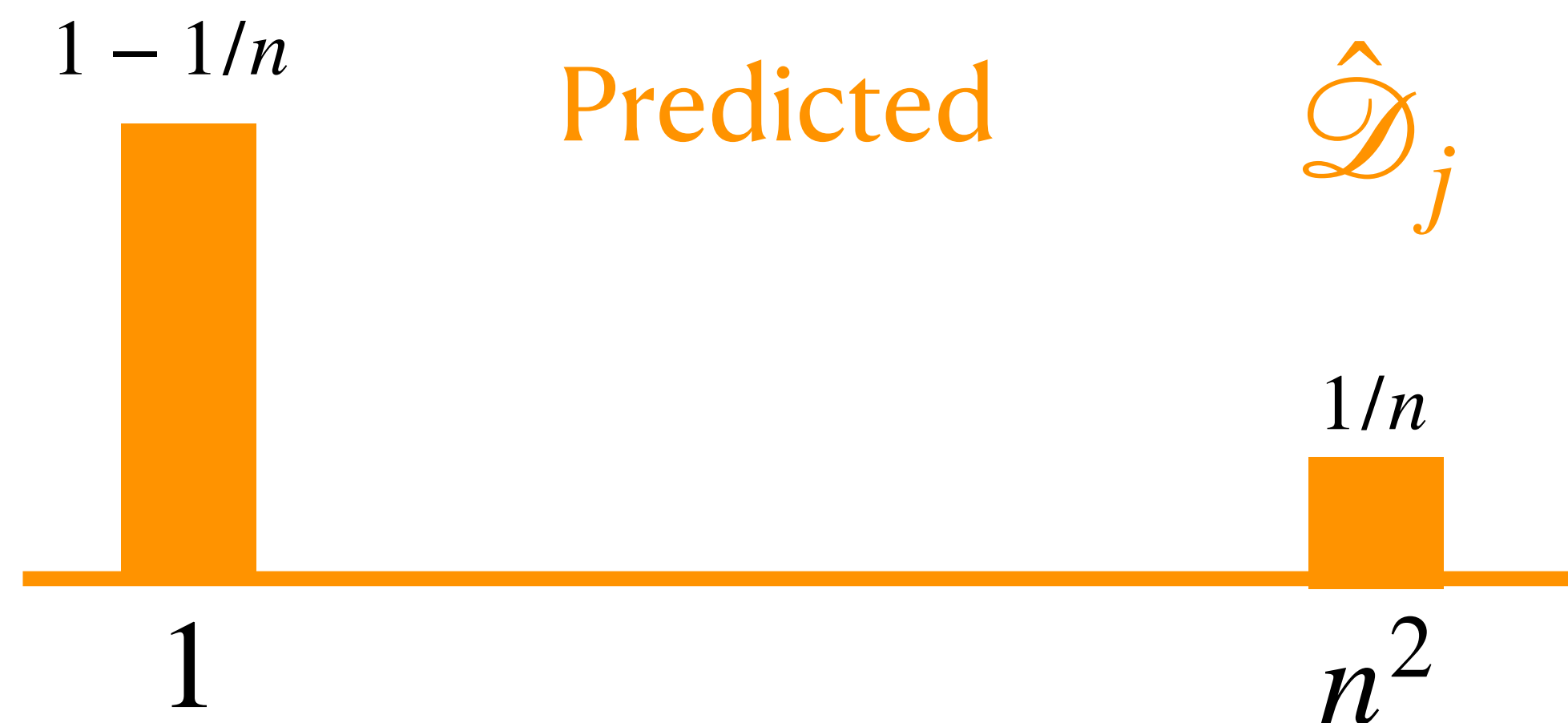
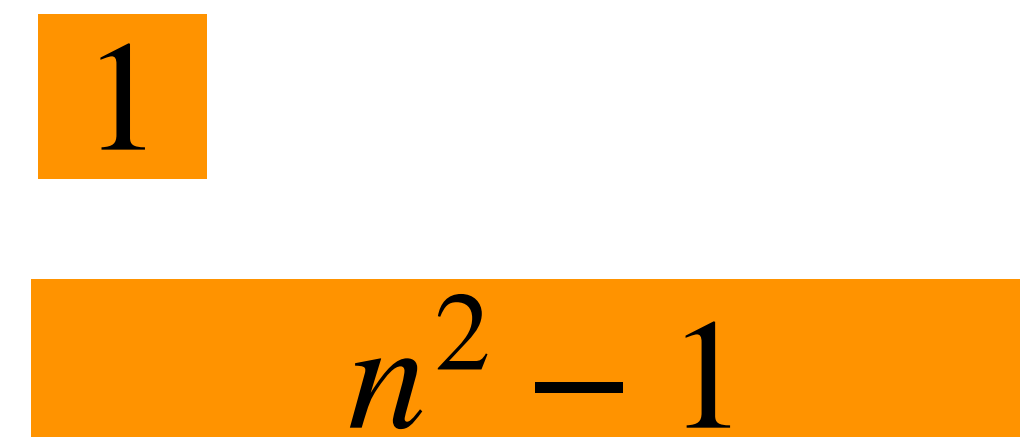
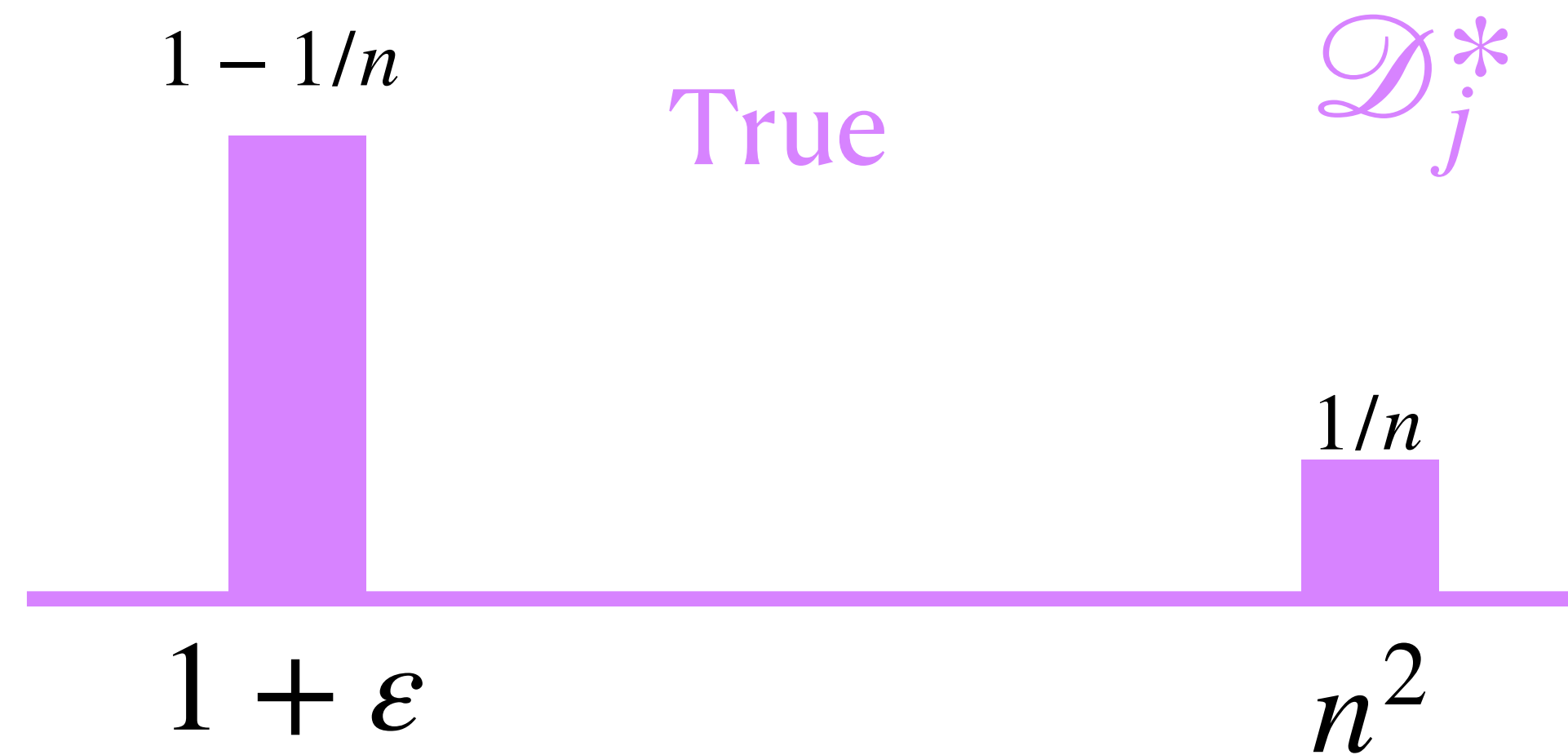
n i.i.d. distributions



Gittins is Not Robust

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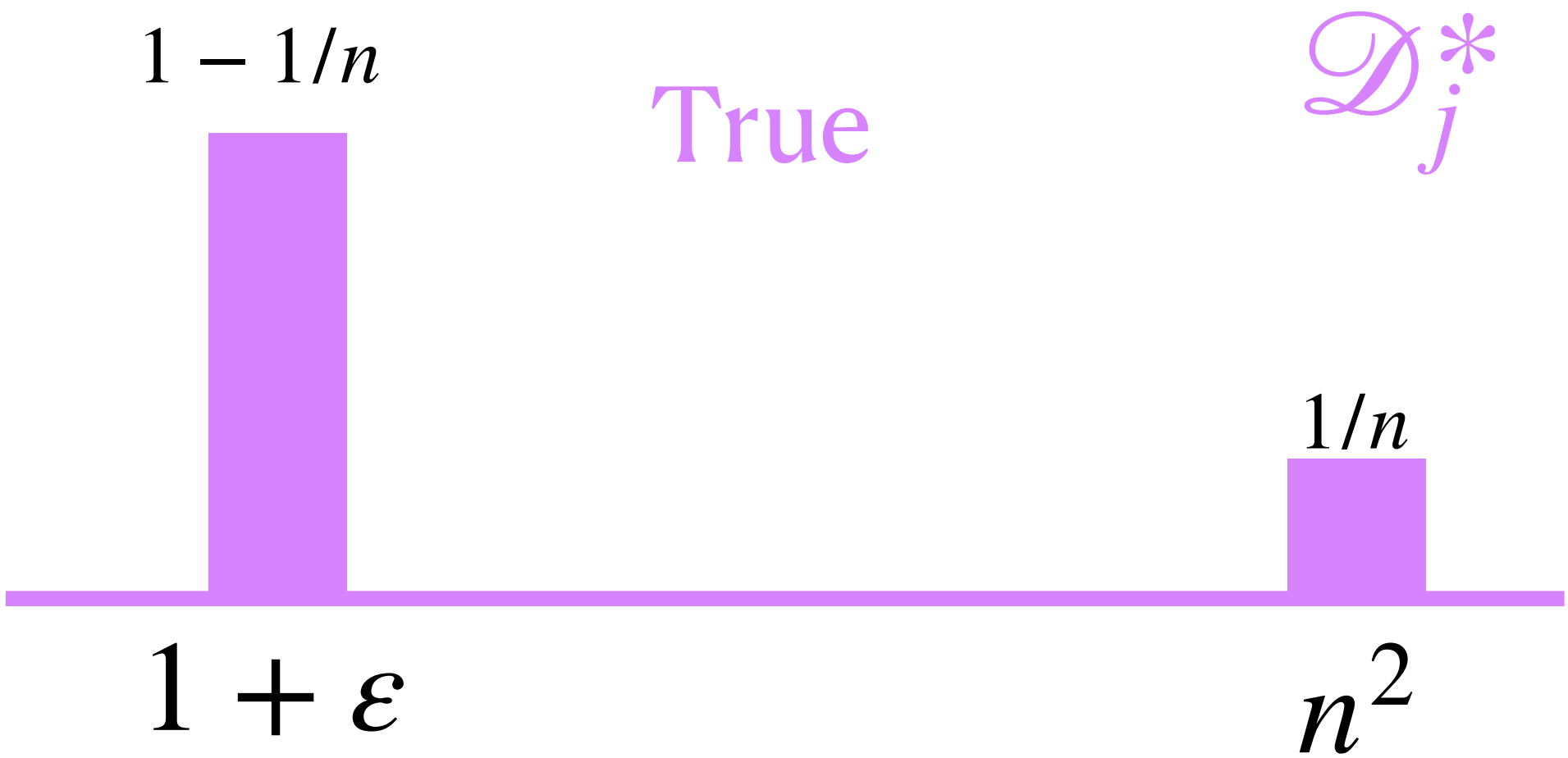
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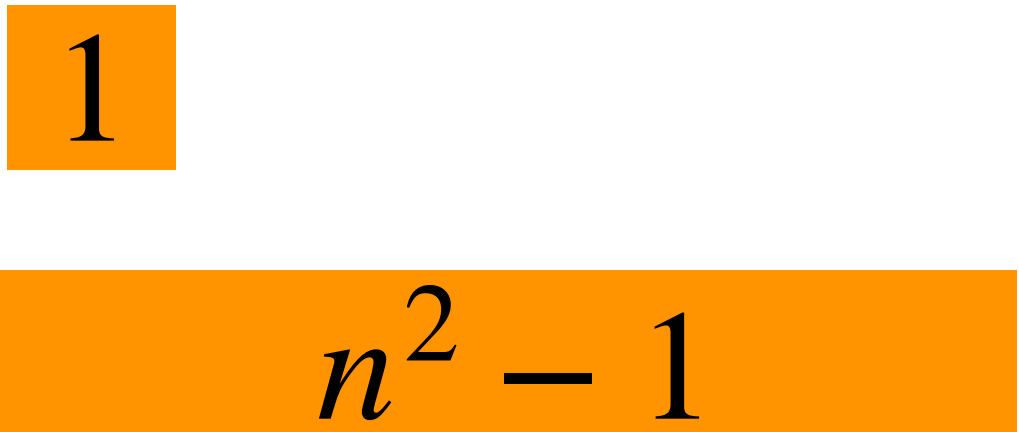
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GIPP

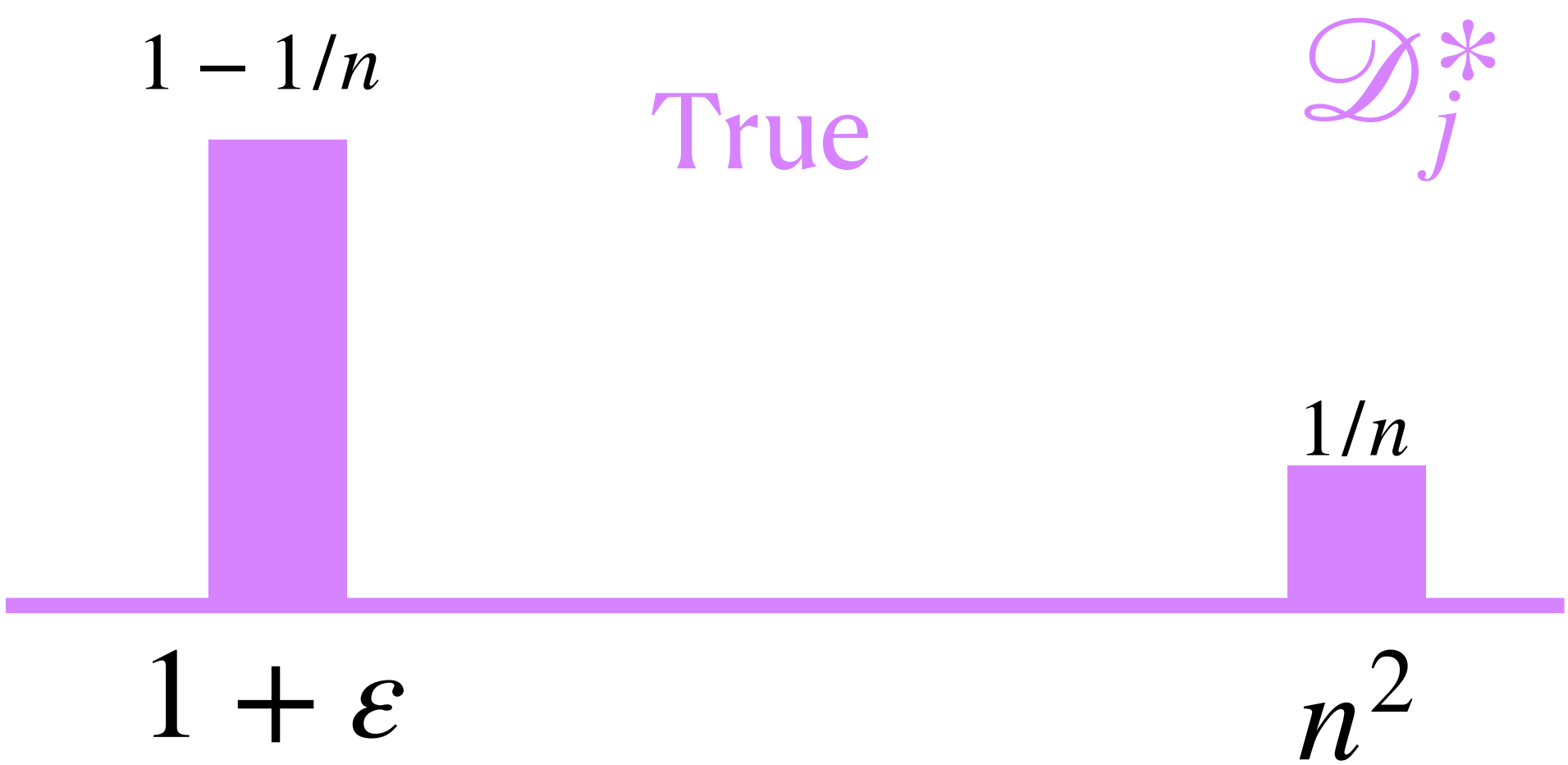


time

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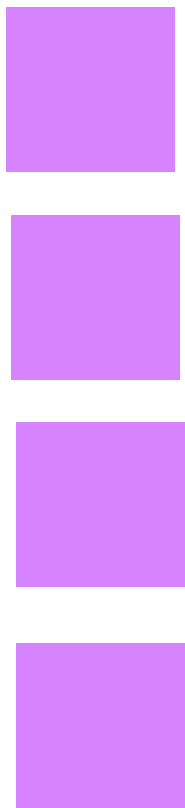


GIPP



1

$n^2 - 1$

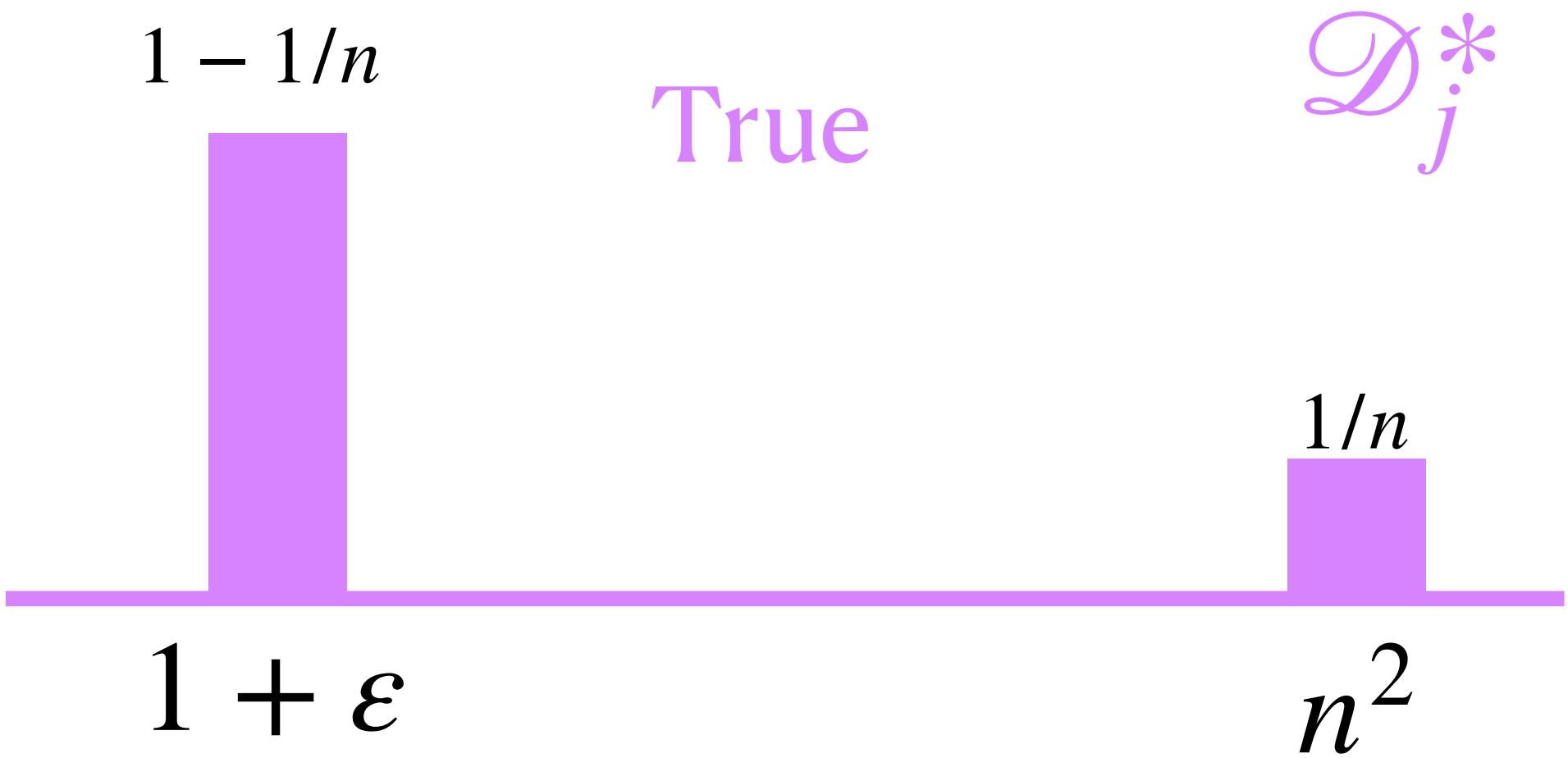


time

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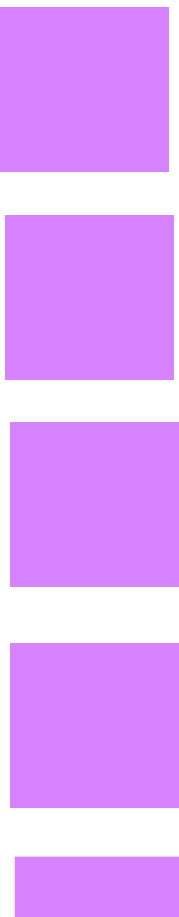


GIPP



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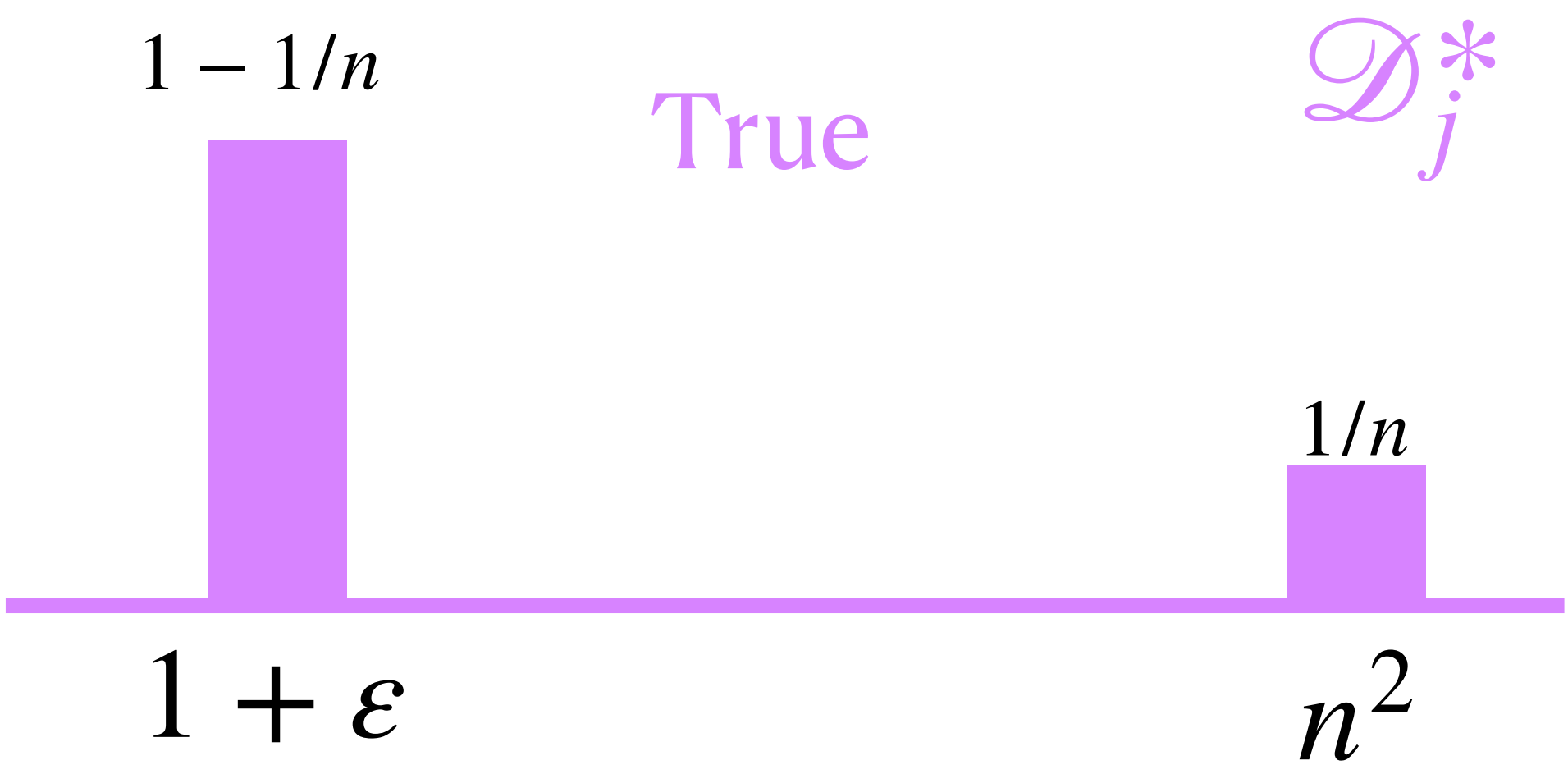
time



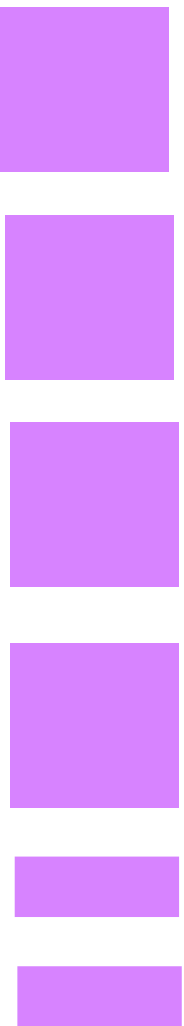
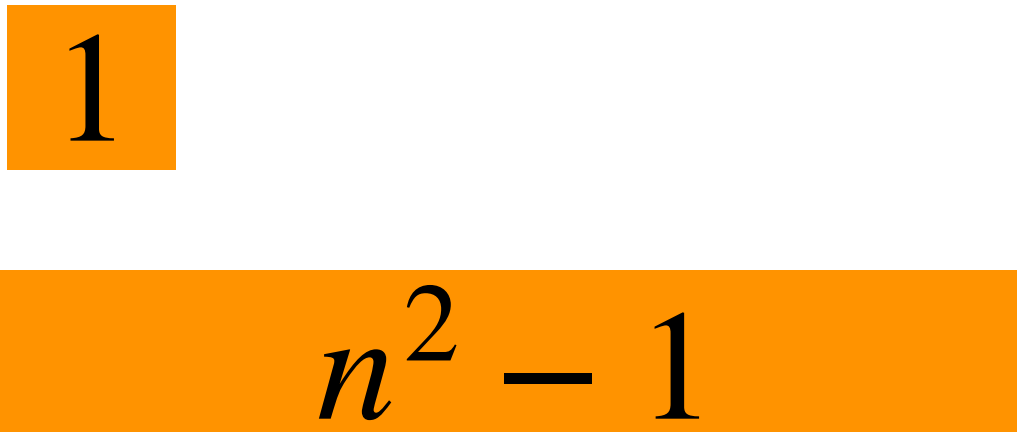
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n i.i.d. distributions



GIPP

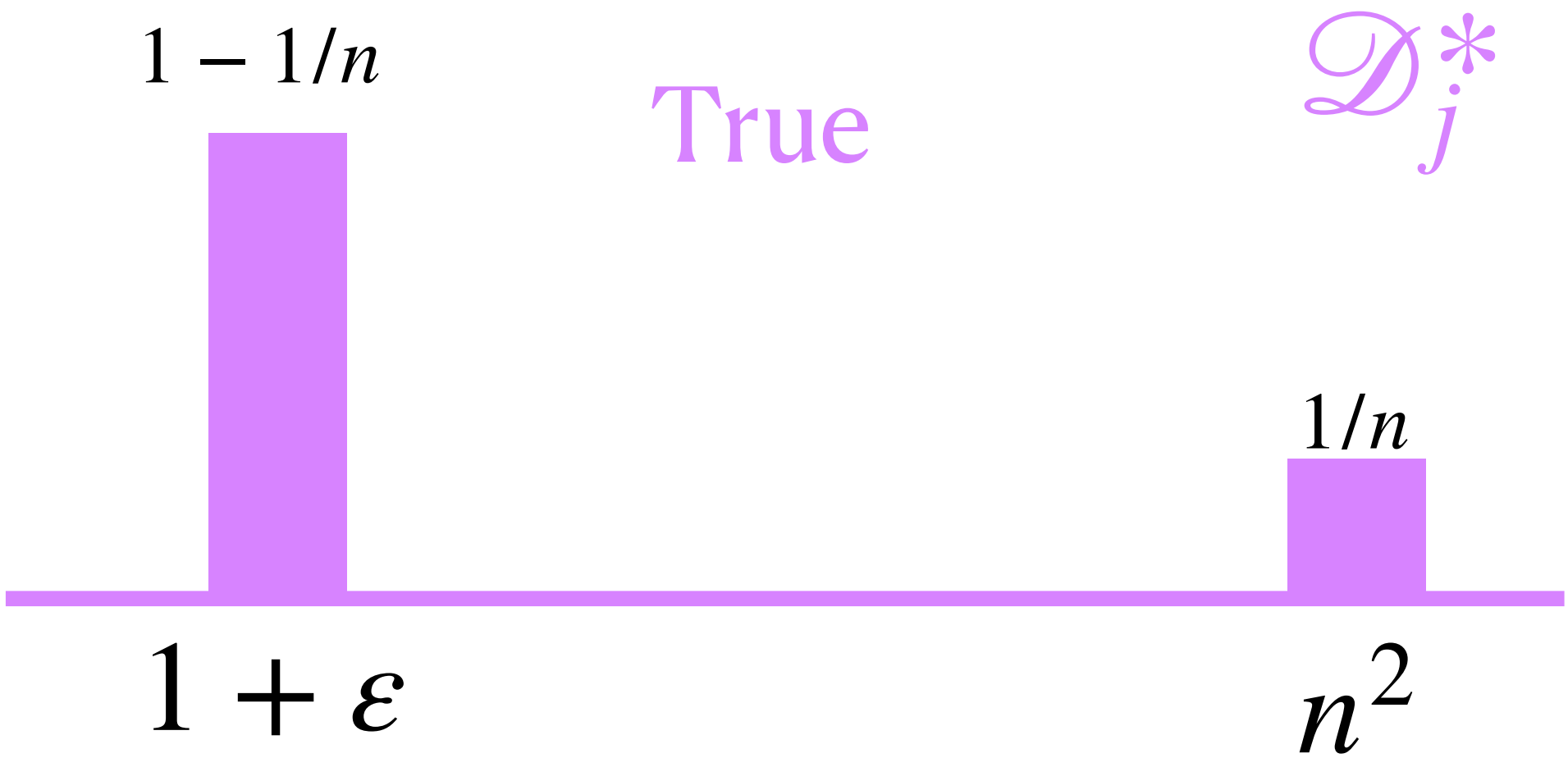


time

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n i.i.d. distributions



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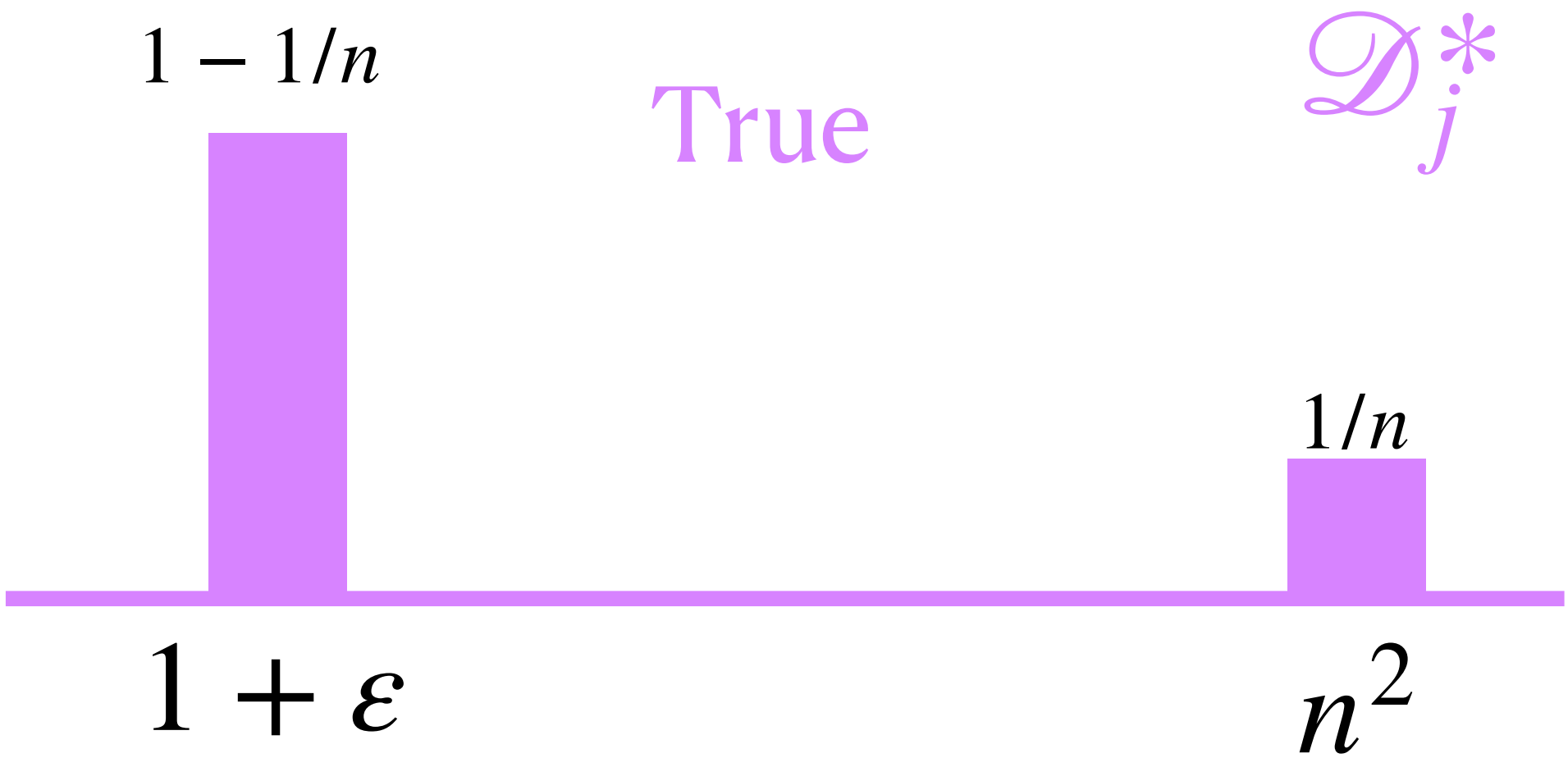
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n i.i.d. distributions



GIPP



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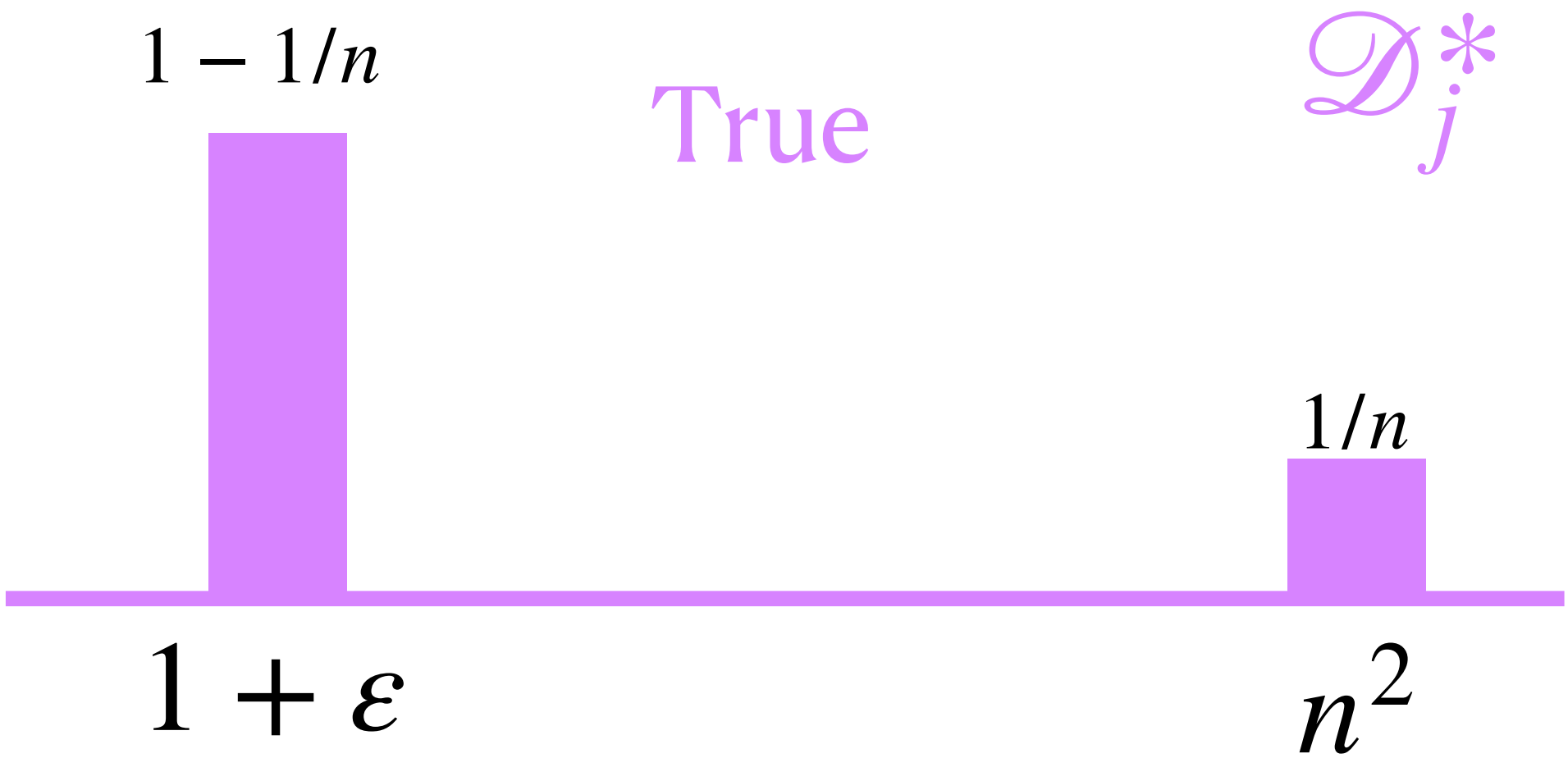


time

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n i.i.d. distributions



GIPP



1

$n^2 - 1$

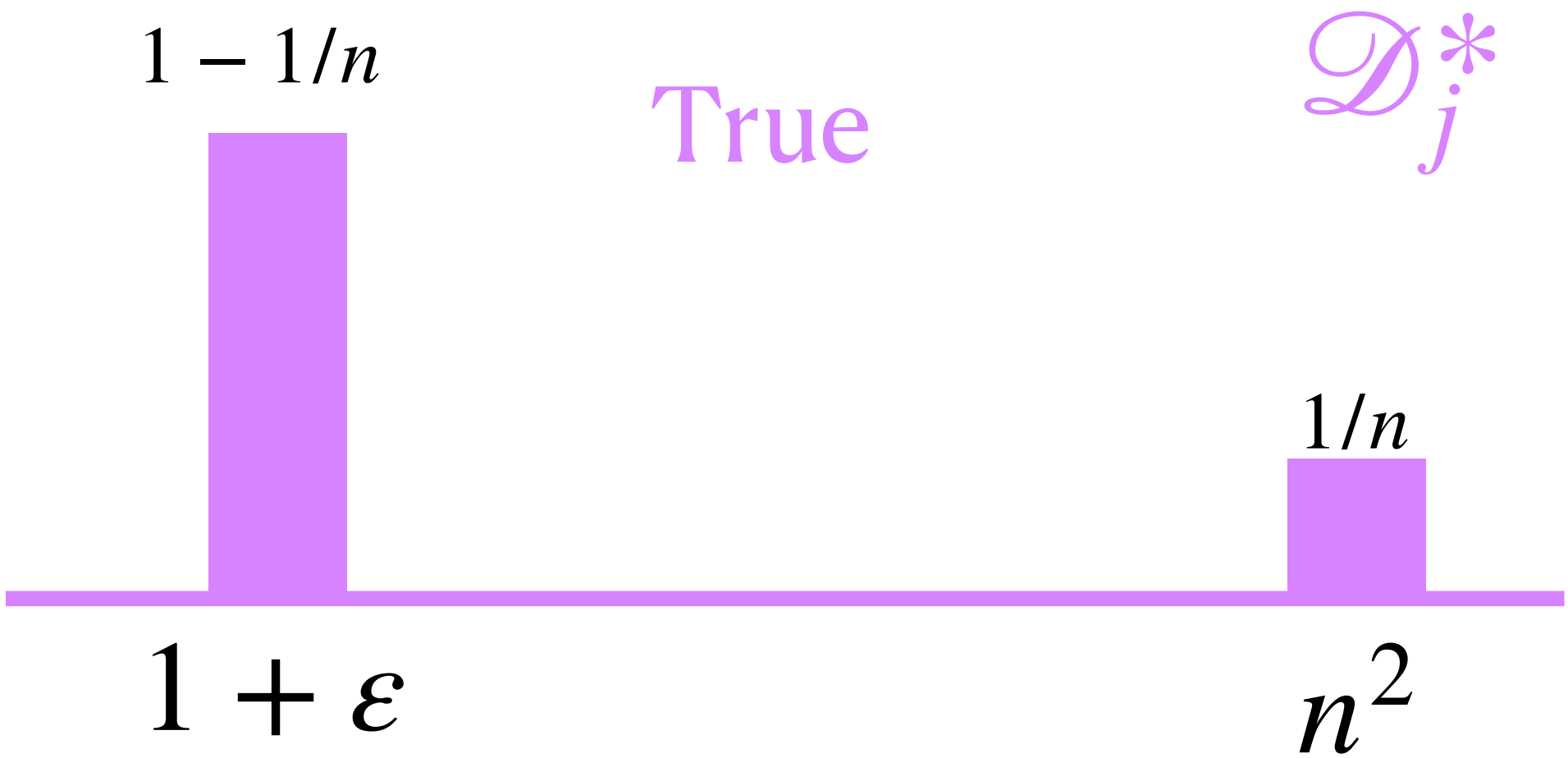


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GIPP

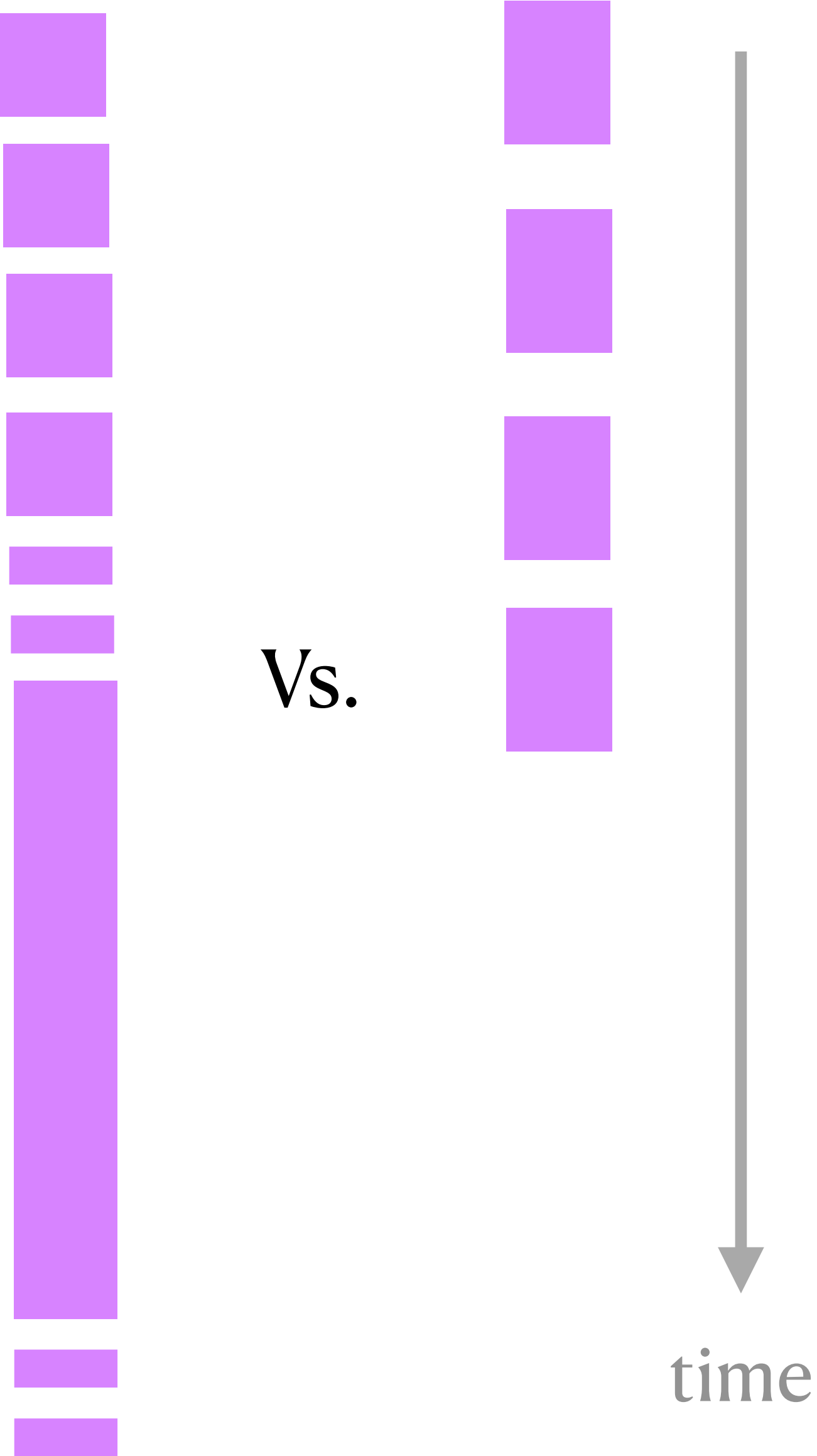


1

$n^2 - 1$



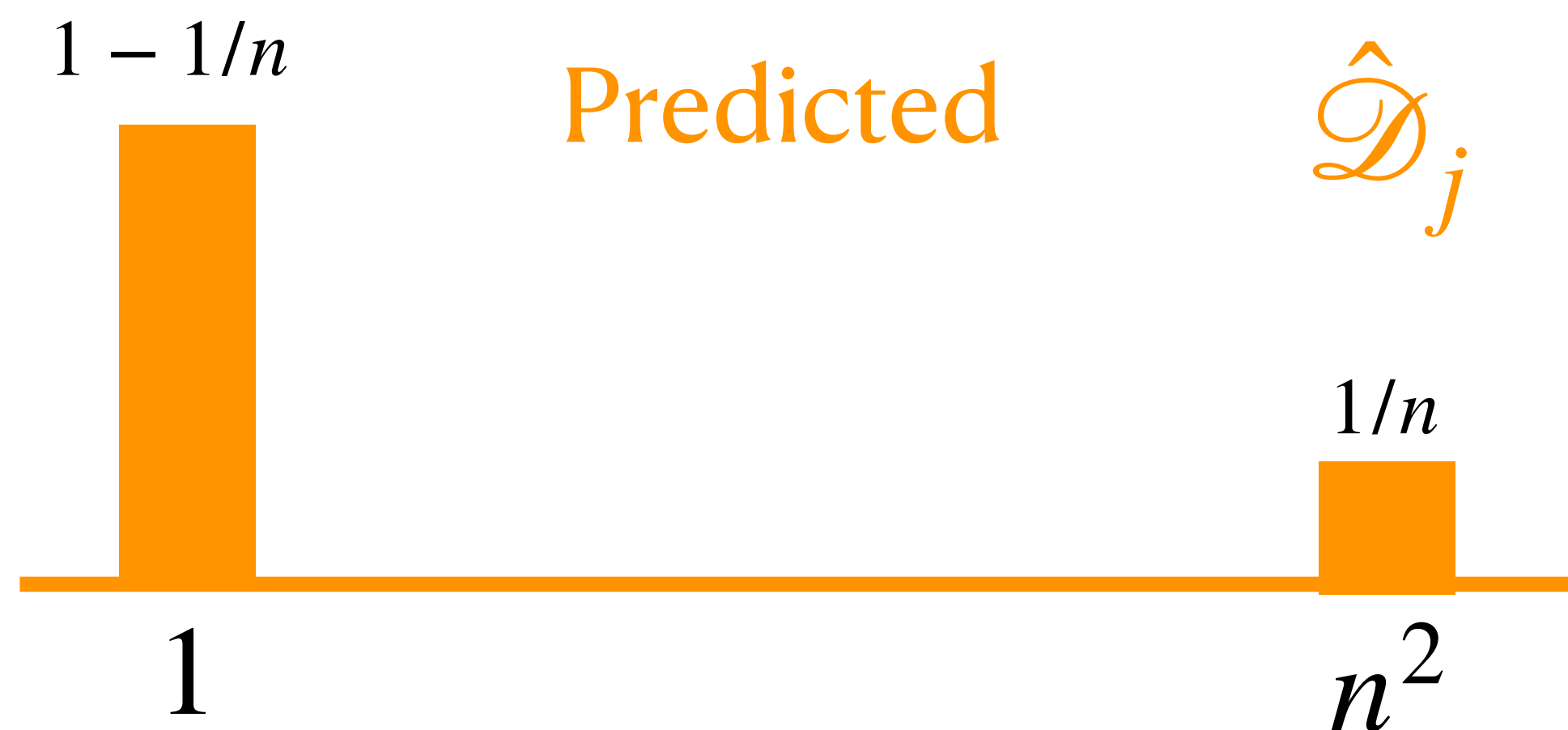
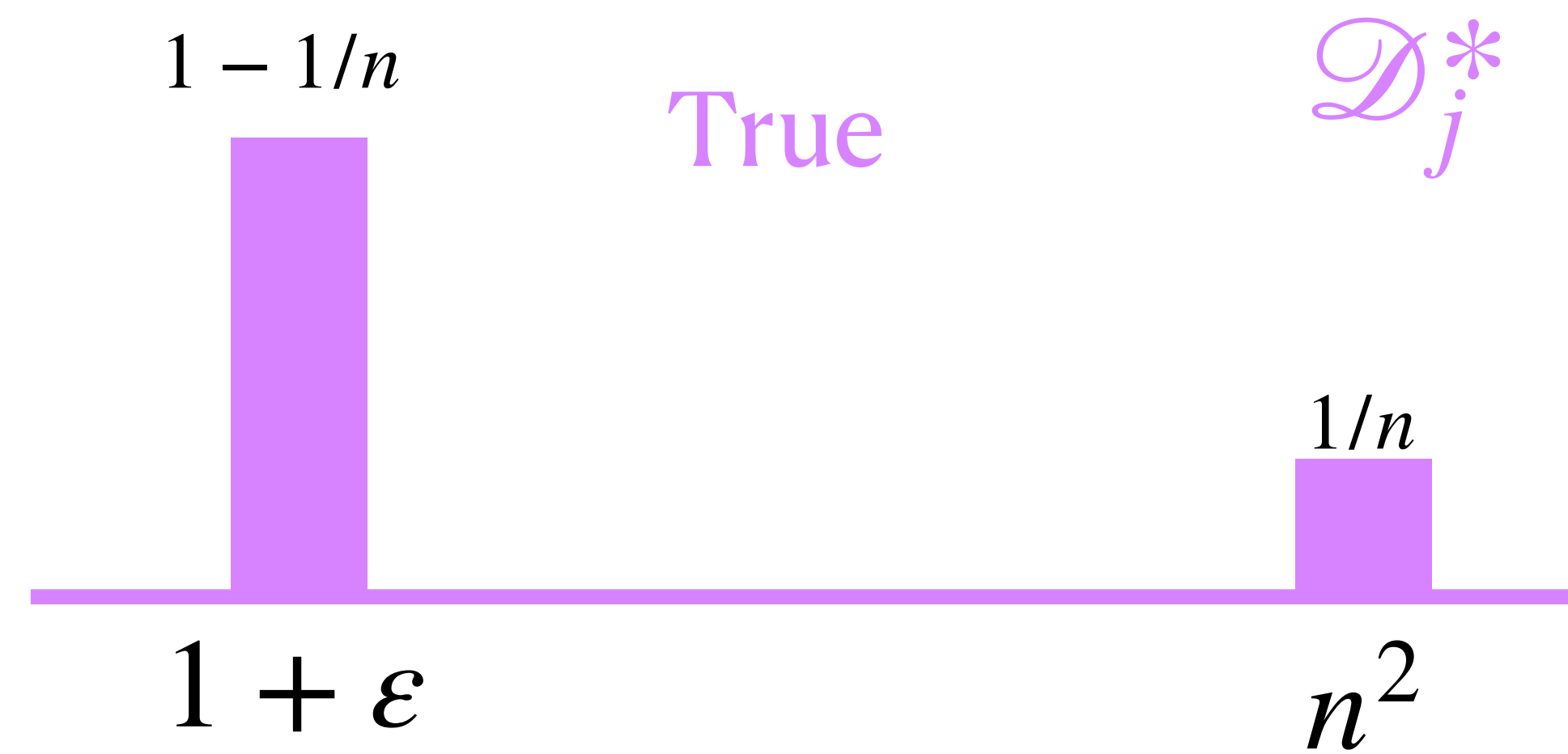
Vs.



Gittins is Not Robust

$$\text{GIPP}(\mathcal{J}^*, \hat{\mathcal{J}}) = \Omega(n) \cdot \text{OPT}(\mathcal{J}^*)$$

n i.i.d. distributions



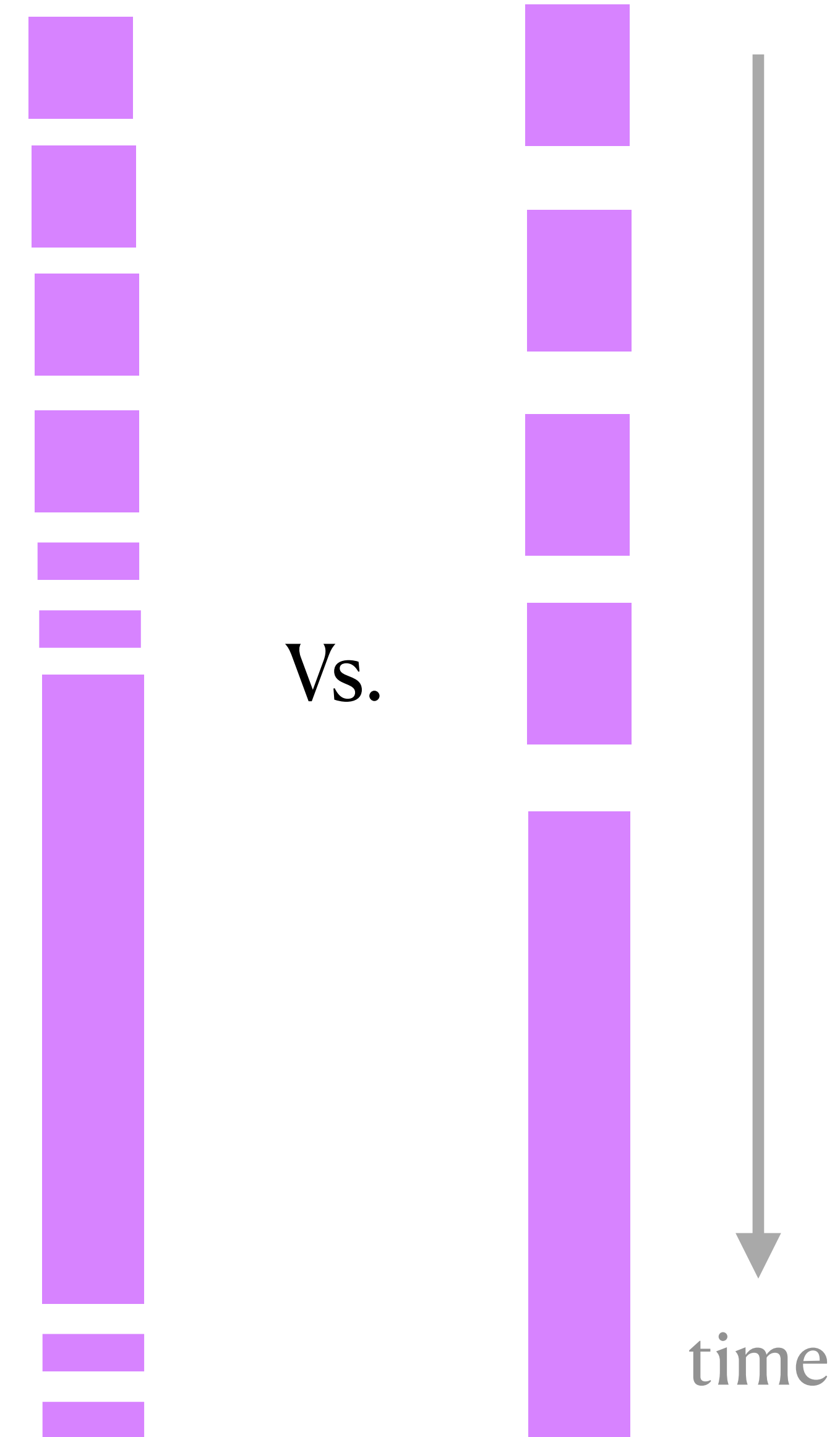
GIPP



1

$n^2 - 1$

Vs.



Robust Gittins (RG)

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Theorem. Given an “ α -close” ($\mathcal{J}^* = \{\mathcal{D}_j^*\}_{j=1}^n, \hat{\mathcal{J}} = \{\hat{\mathcal{D}}_j\}_{j=1}^n$), $\alpha \geq 1$, pair of true and predicted distributions, there is a policy, Robust Gittins (RG), satisfying:

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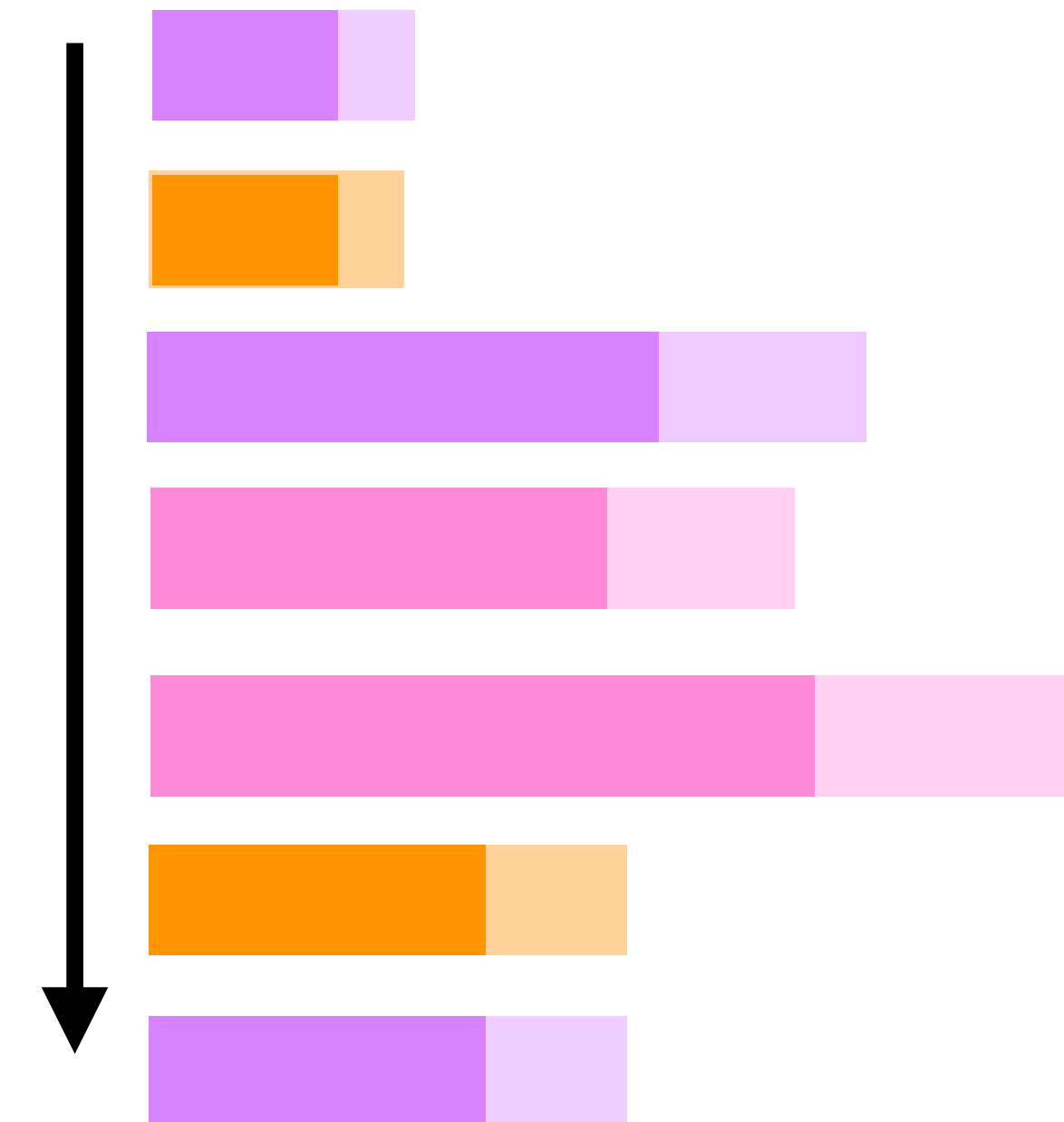
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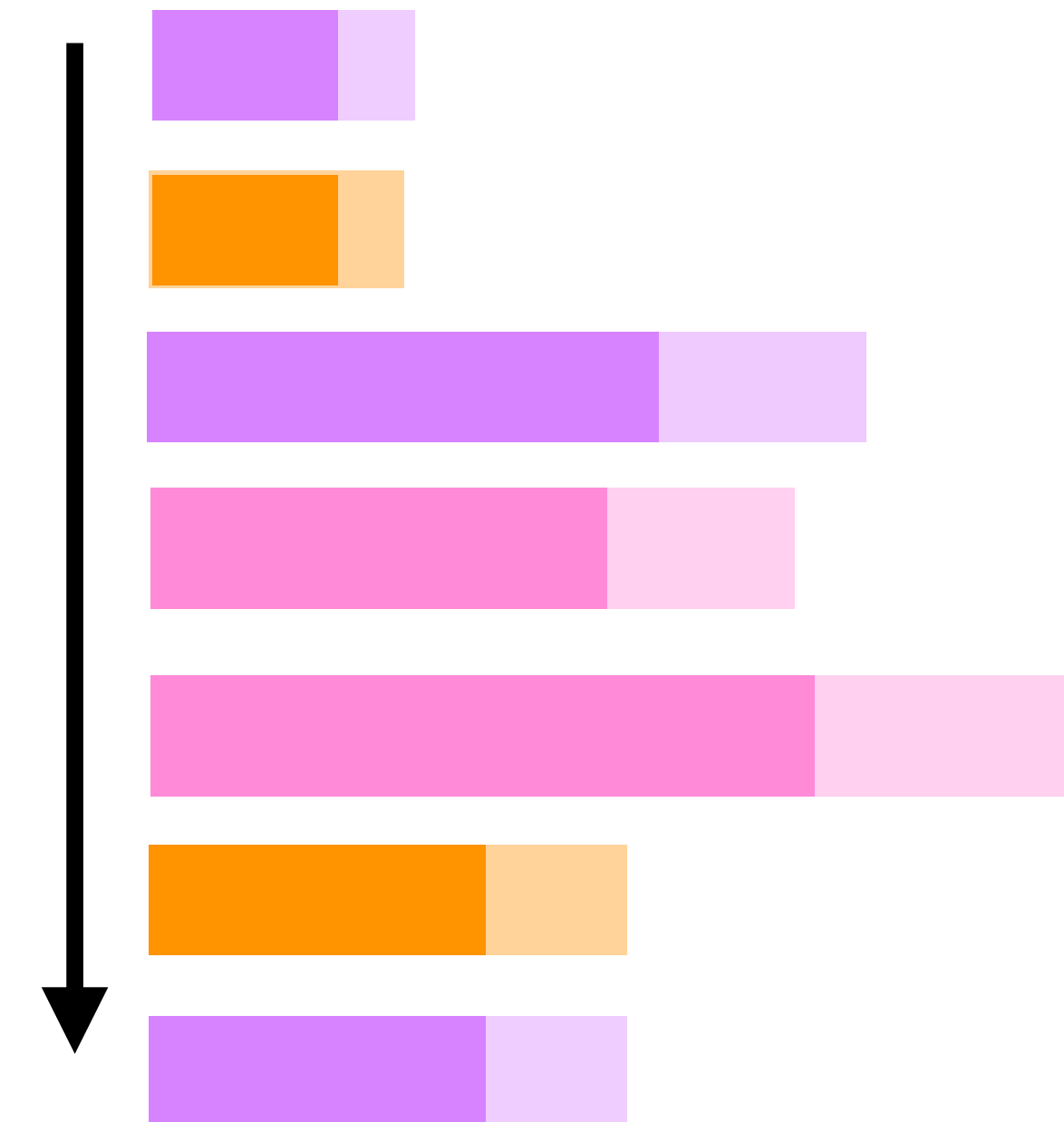
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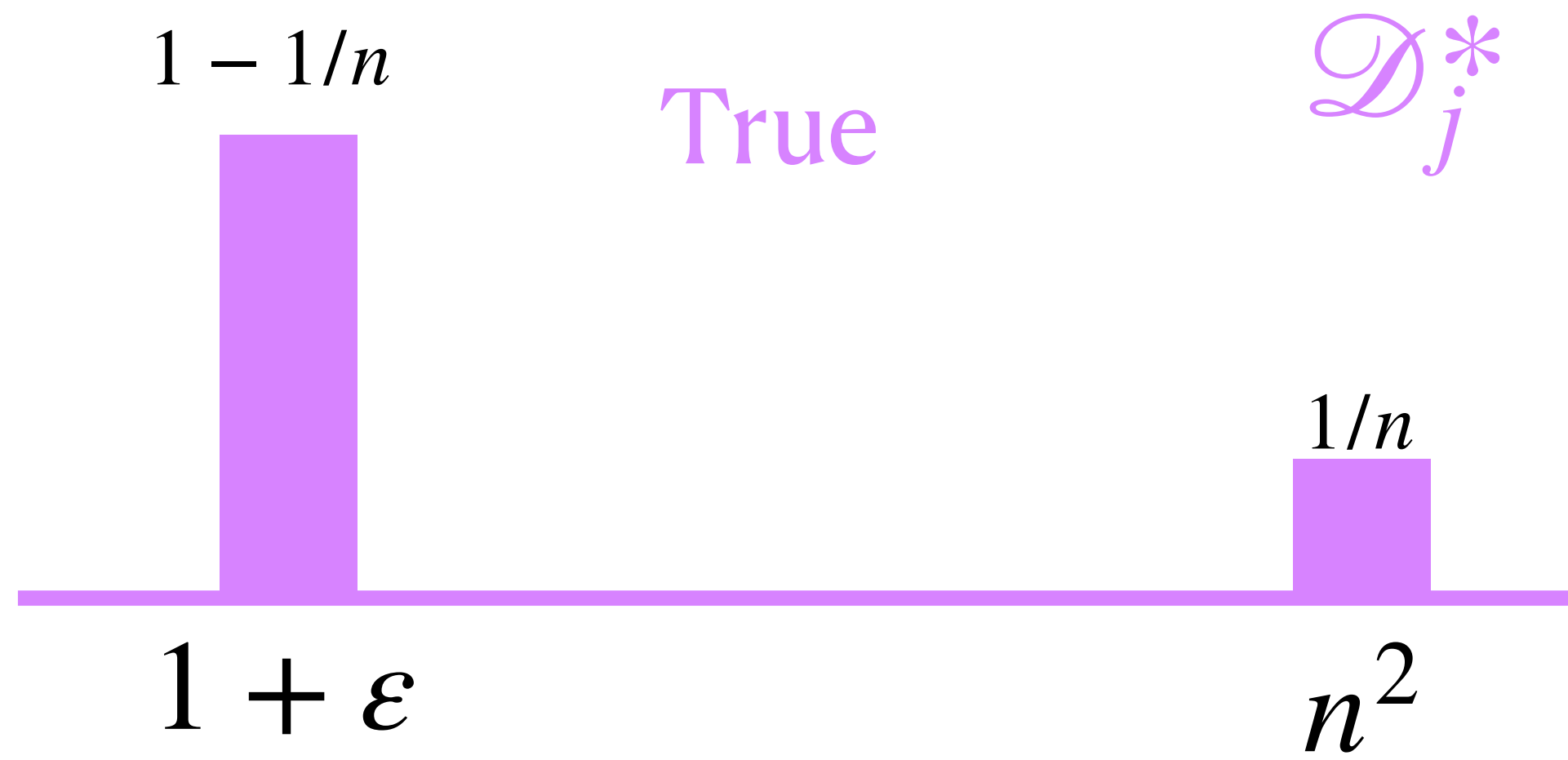
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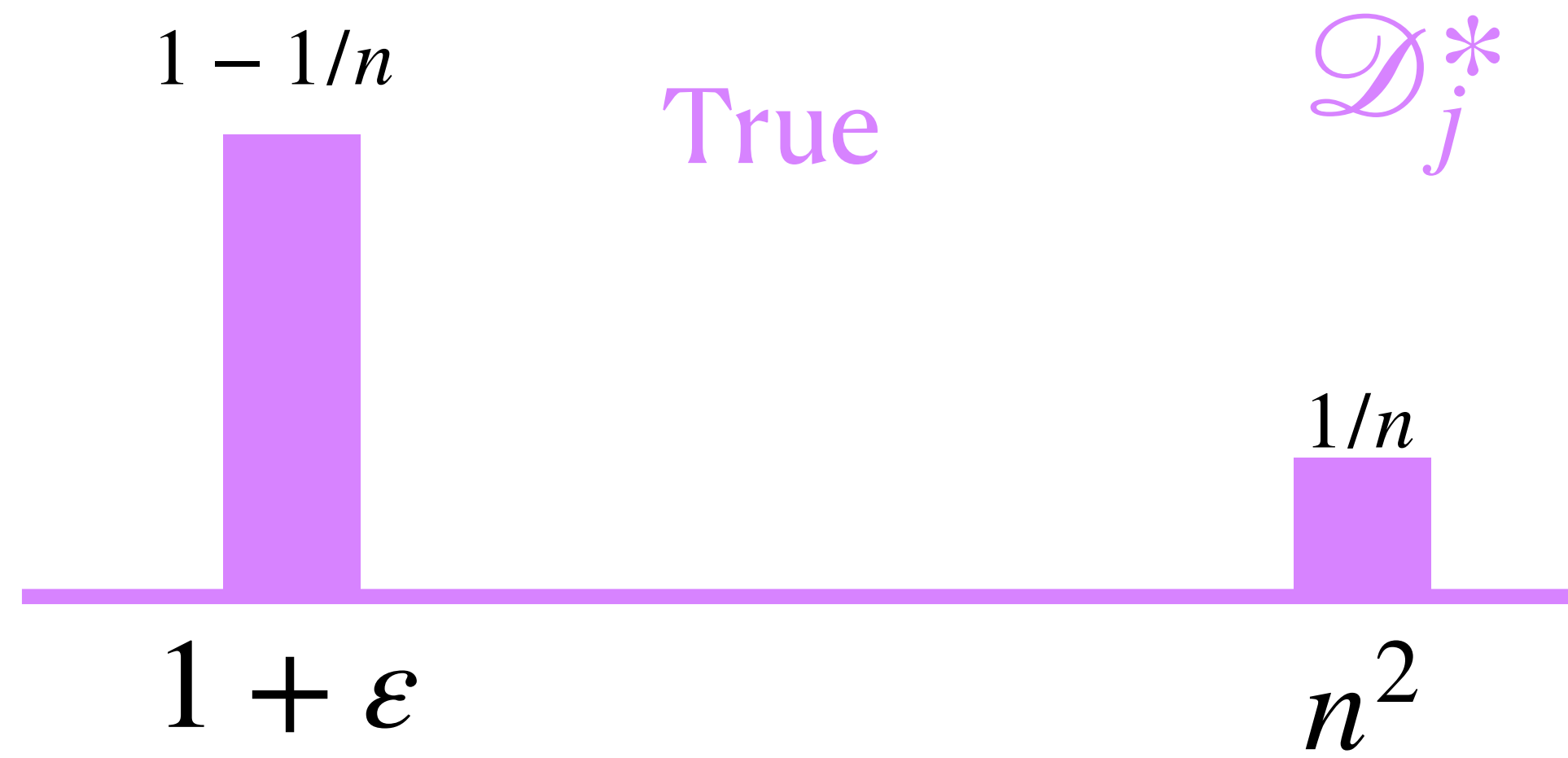
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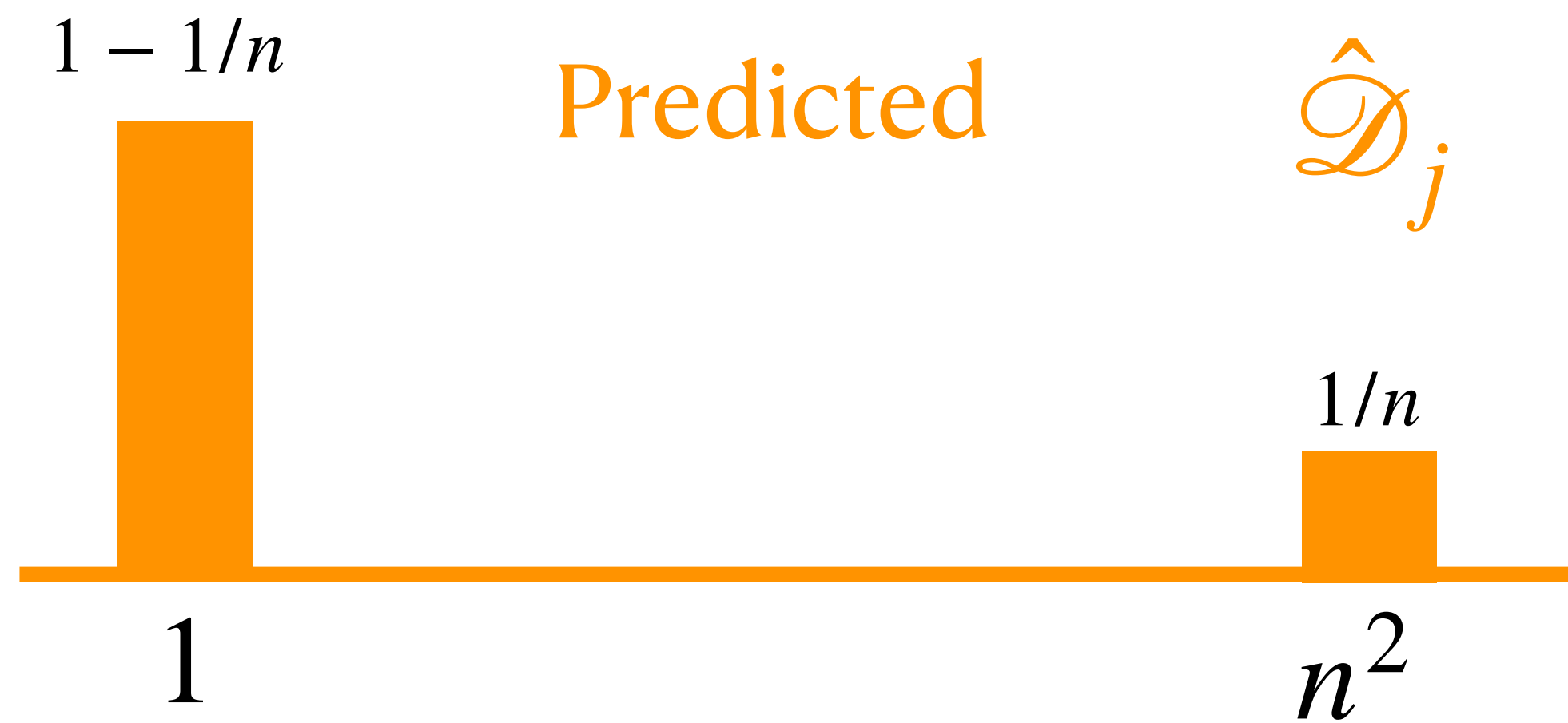
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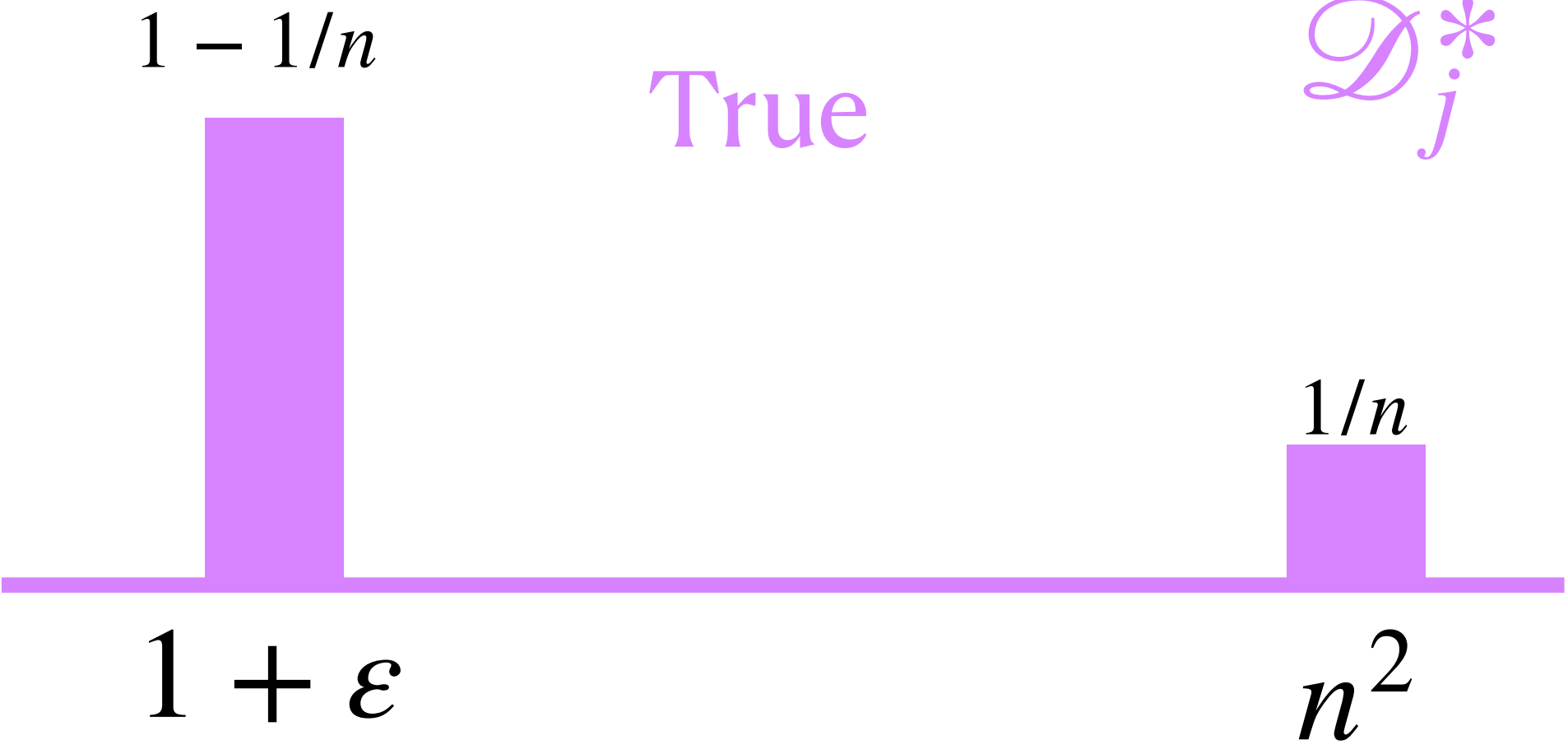
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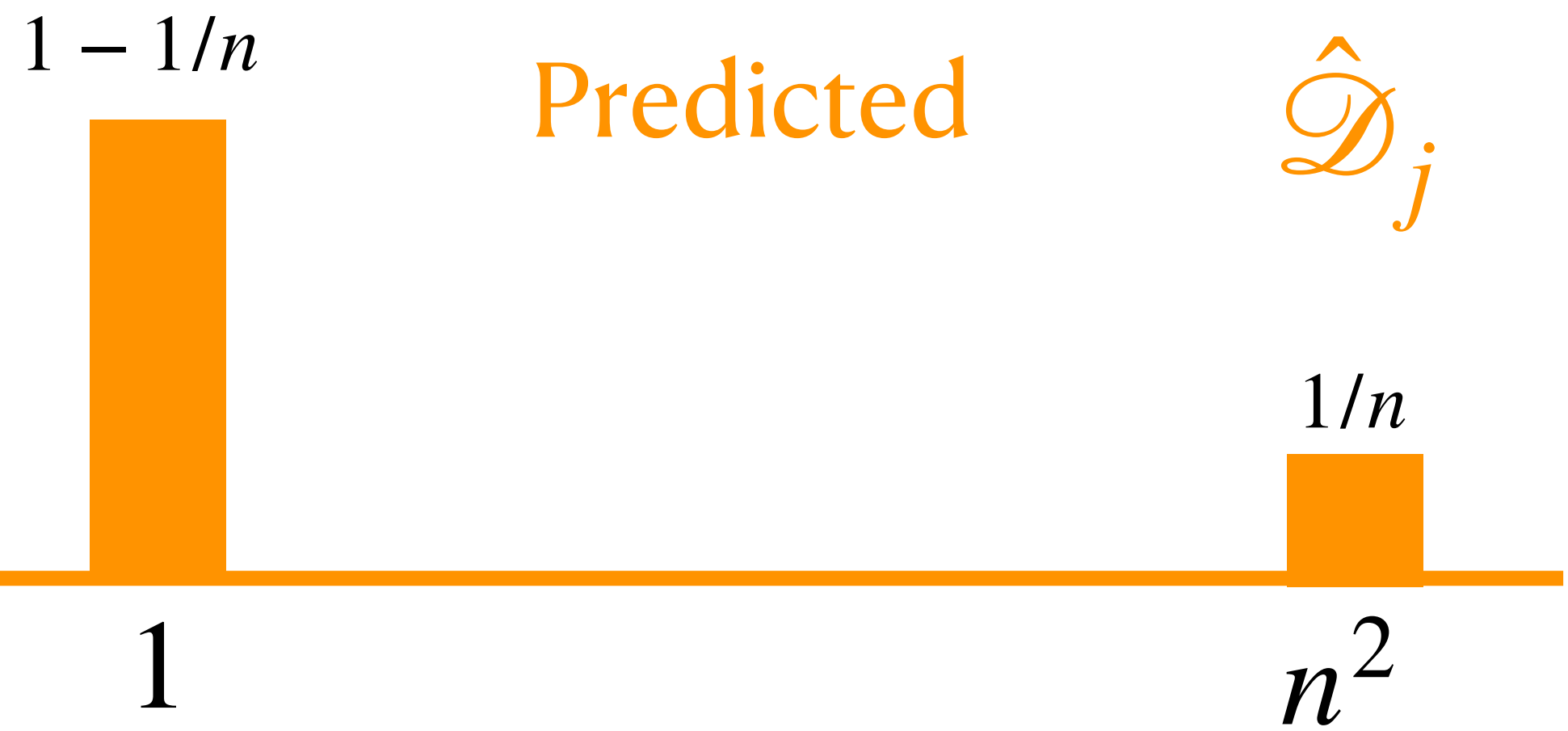
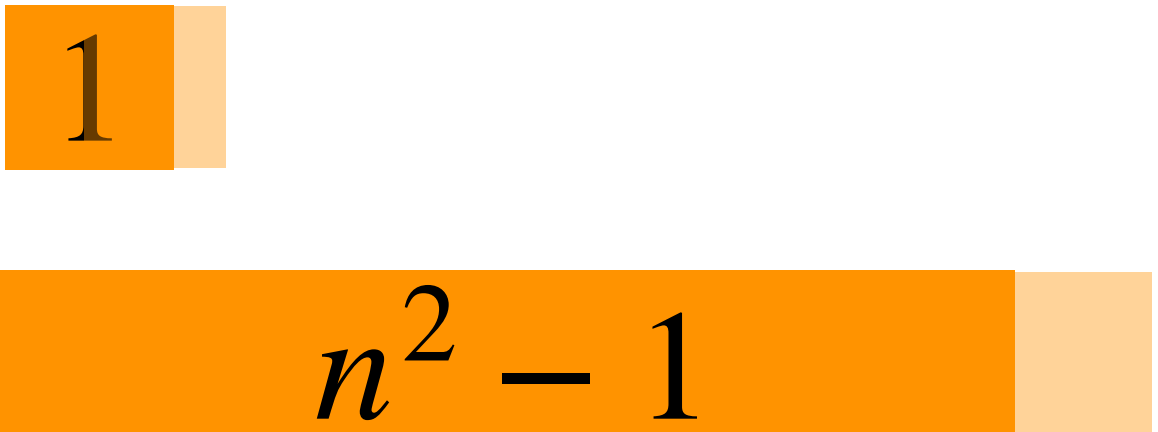
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A New Error Measure

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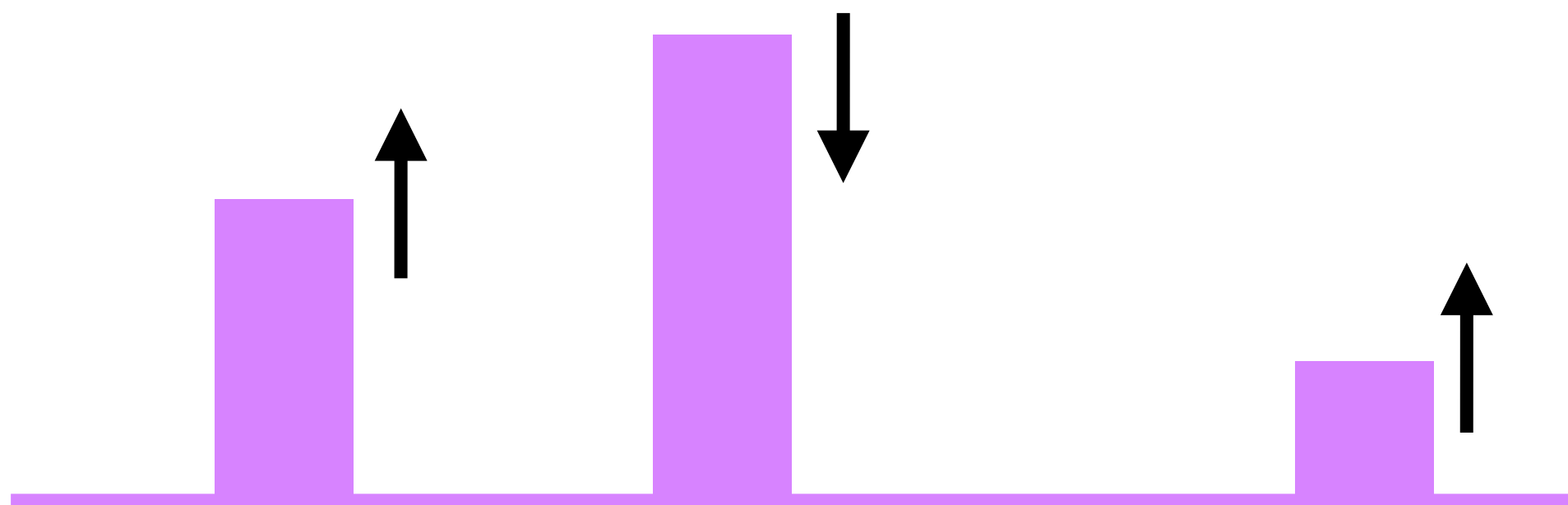
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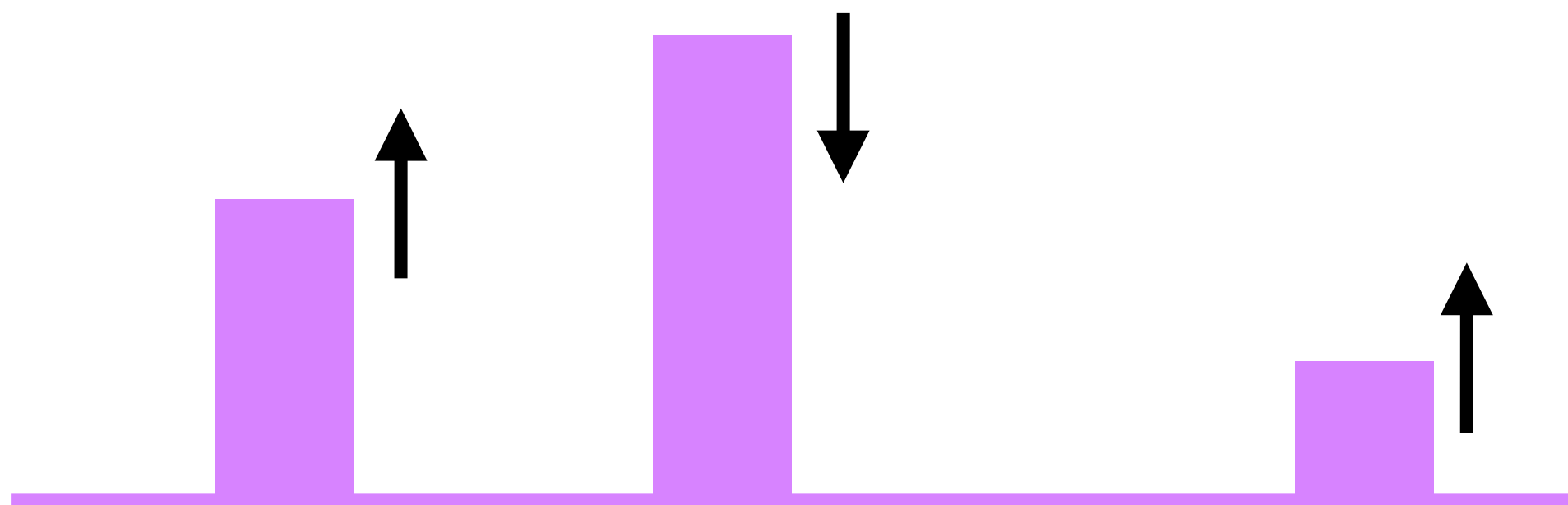
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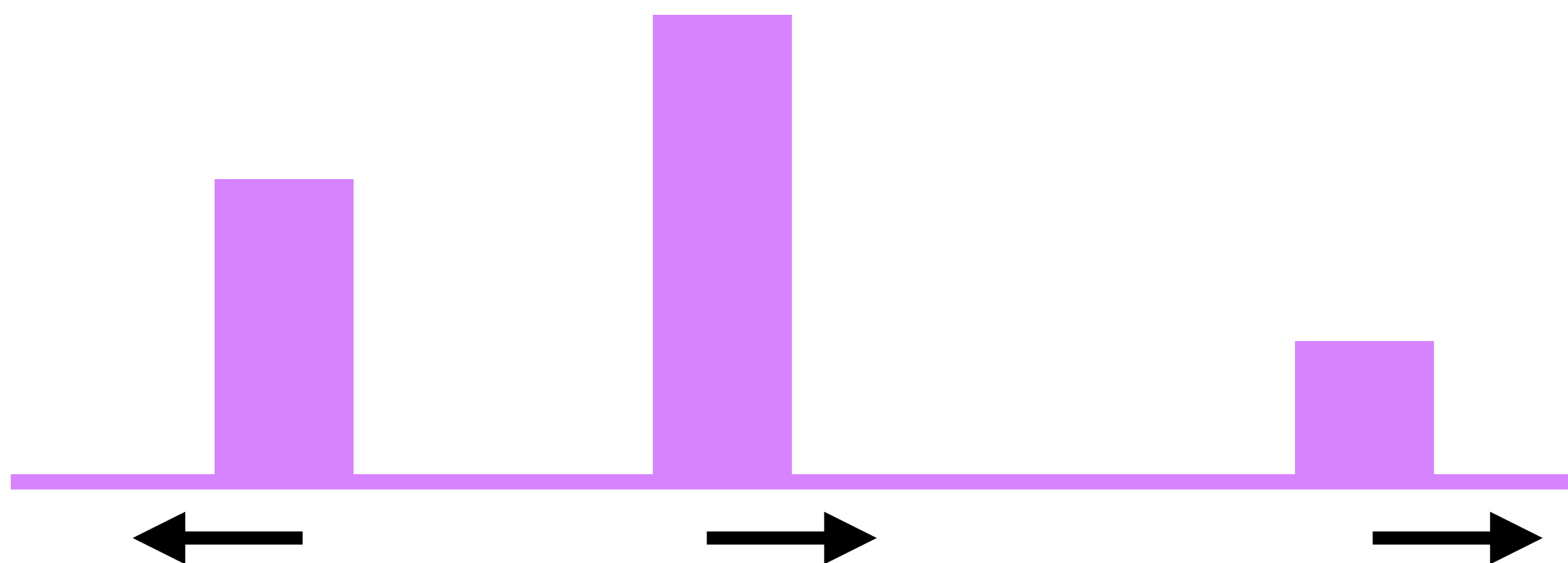
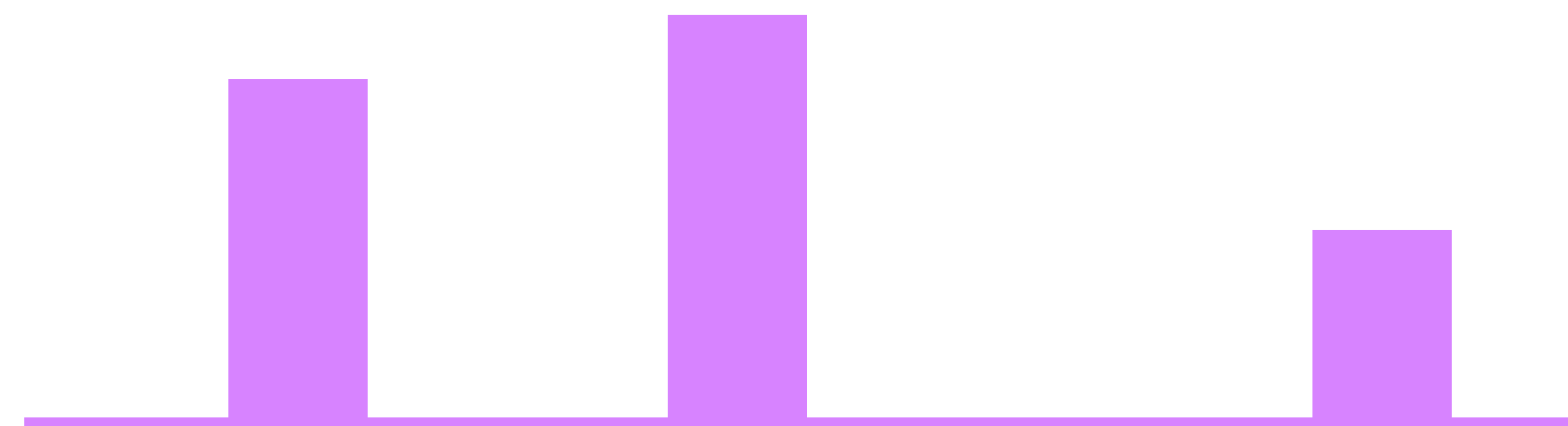
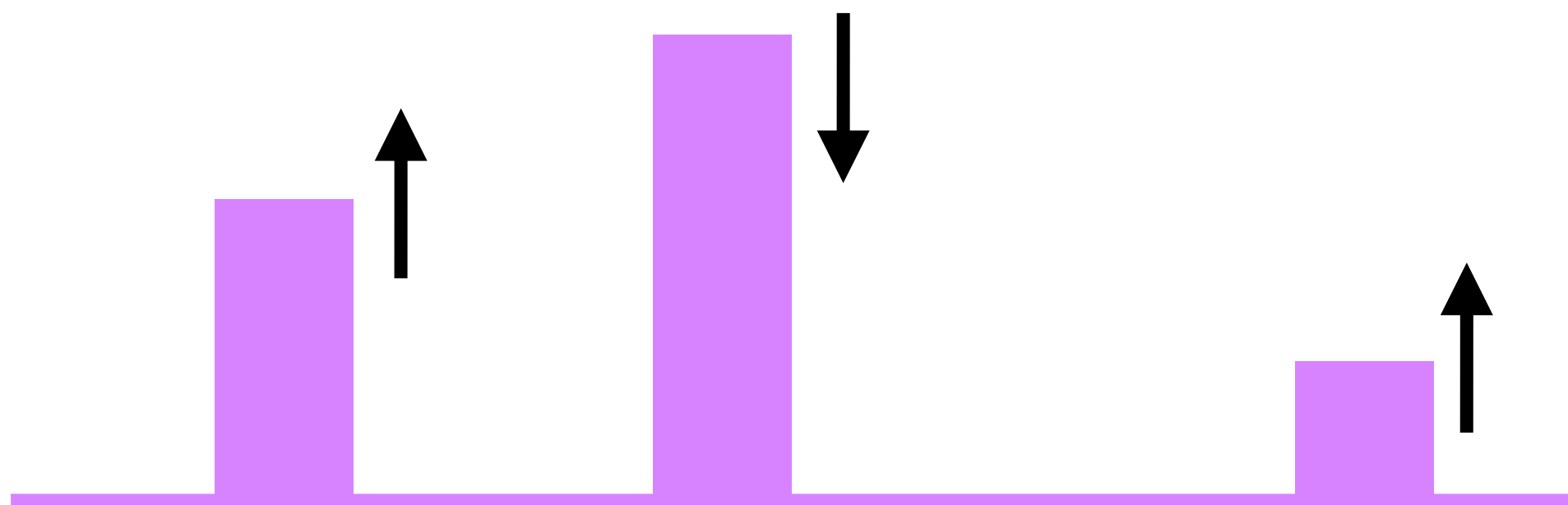
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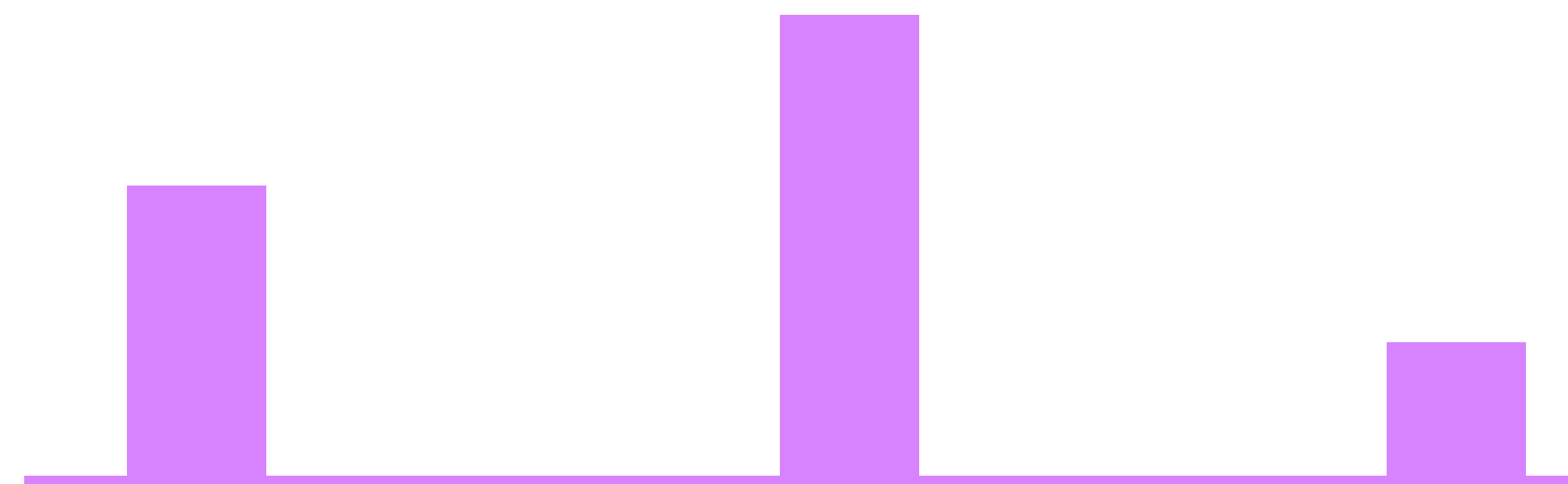
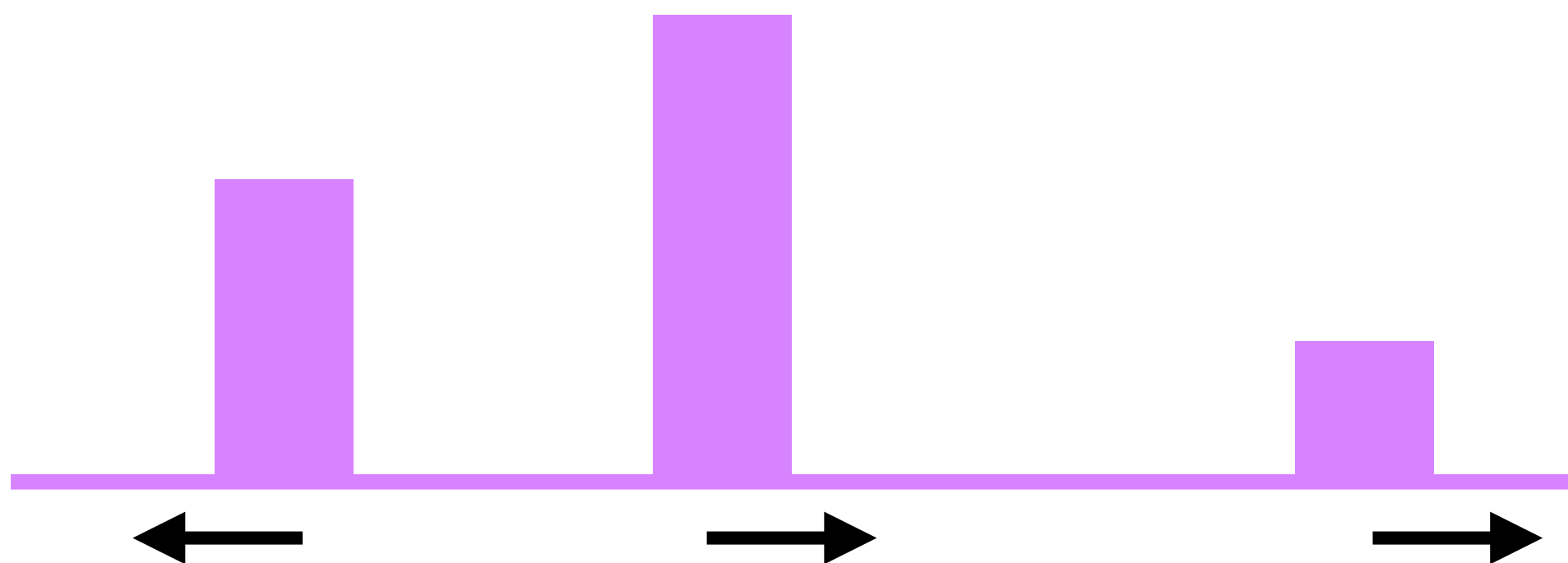
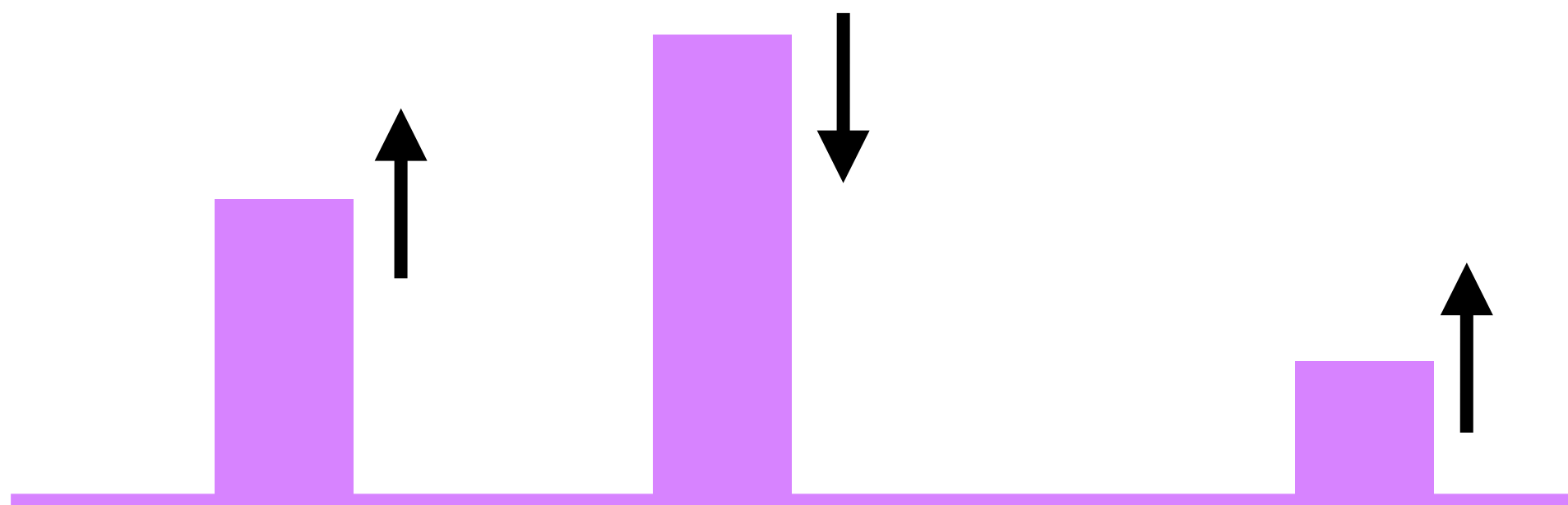
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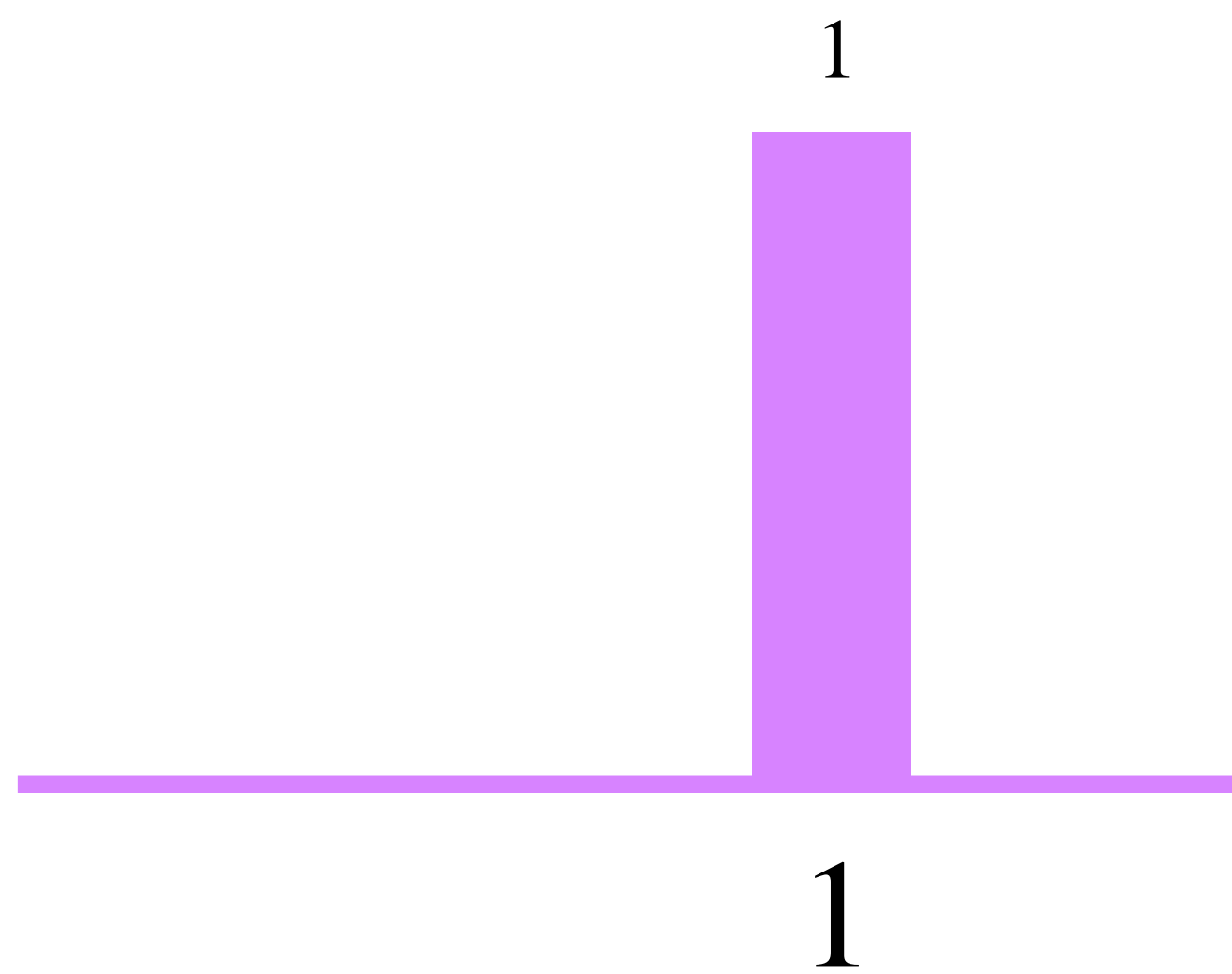
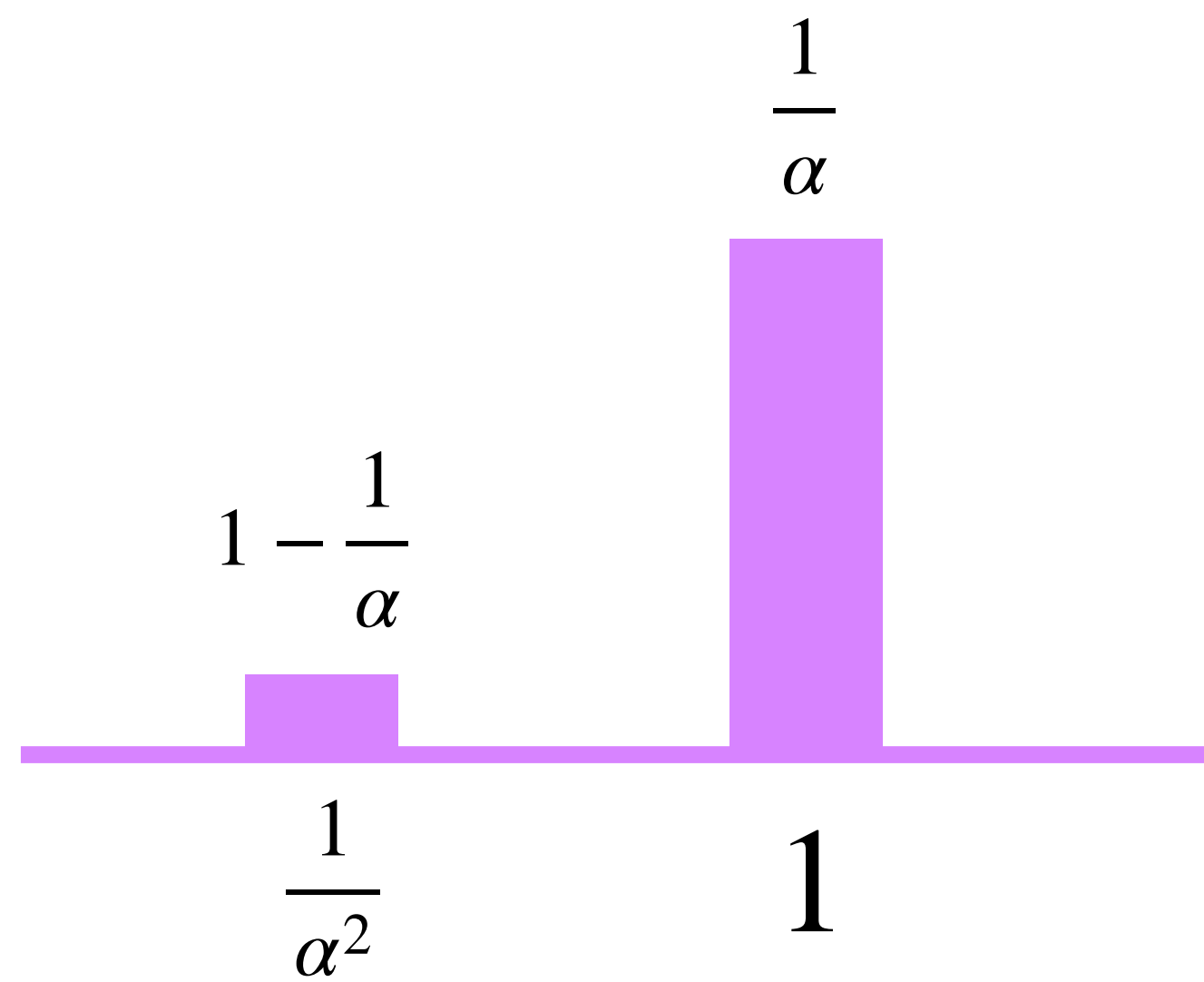
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...and a less immediately obvious example

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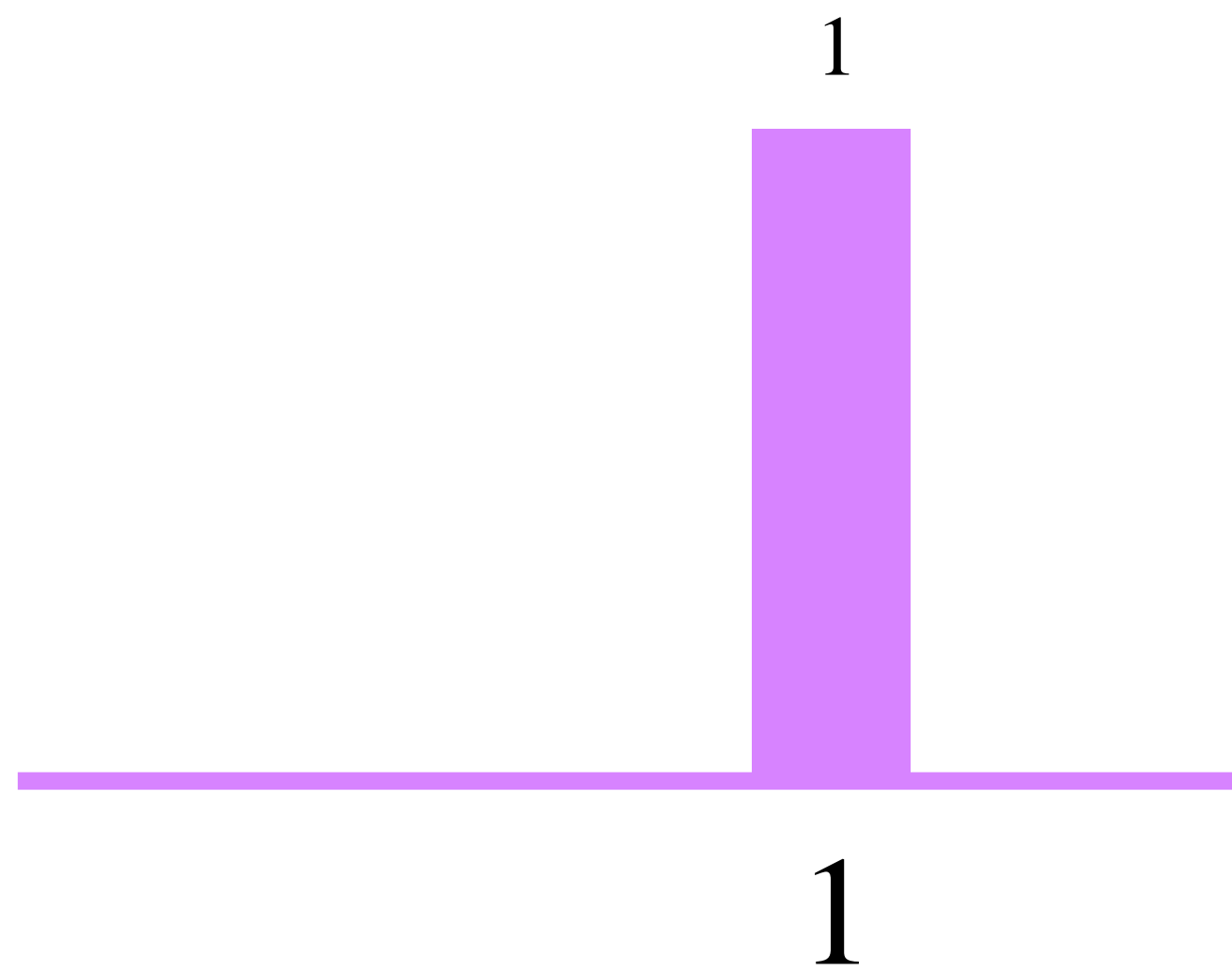
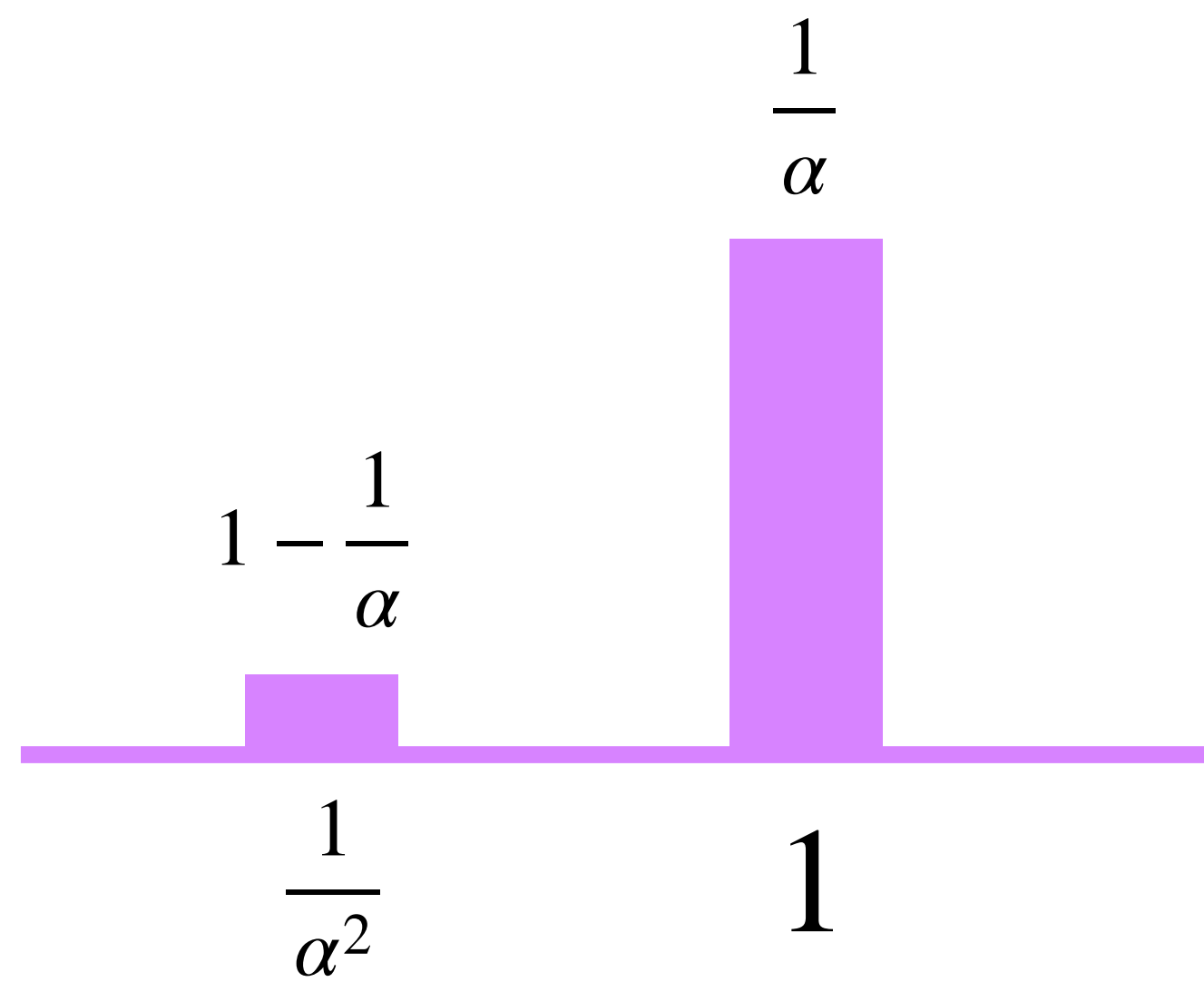
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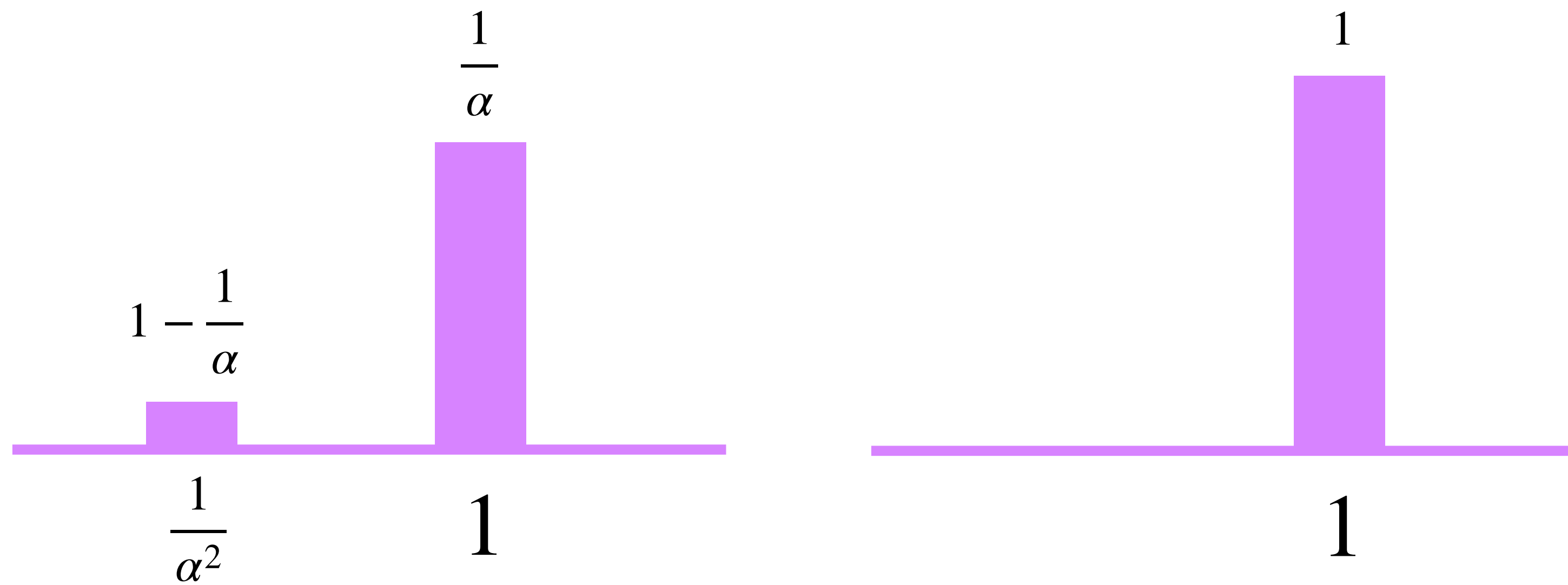


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$Exp(\lambda)$ and $Exp(\lambda')$ are $\frac{\lambda'}{\lambda}$ - close

Relation to Other Distances

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Lévy distance

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Lévy distance

$$d_L(\mathcal{D}, \mathcal{D}') = \inf\{\varepsilon \geq 0 \mid F'(x - \varepsilon) - \varepsilon \leq F(x) \leq F'(x + \varepsilon) + \varepsilon \quad \forall x \in \mathbb{R}\}$$

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Note: proves $\text{OPT}(\hat{\mathcal{J}})$ and $\text{OPT}(\mathcal{J}^*)$ off by factor of at most α^3

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Closed-form cost of Gittins (Megow & Vredeveld '14)

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$$\text{GIPP}(\mathcal{J}, \mathcal{J}) = \sum_{j=1}^n \sum_{i=1}^{n_j} \sum_{(k, q_{k,l}) \in H'(j,i)} \mathbb{E} \left[\mathbf{1}_{\{P_j > y_{j,i}\}} \cdot \mathbf{1}_{\{P_k > y_{k,l}\}} \cdot \min\{P_k - y_{k,l}, q_{k,l}\} \right]$$

Robust Gittins (RG) Analysis

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Upshot: new stochastic scheduling model robust to errors in predicted distributions

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- ✓ Unknown α ?
- ✓ Other problems where (something like) Gittins index is optimal?